KABARAK UNIVERSITY

EXAMINATIONS

2008/2009 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **PHYS 311**

COURSE TITLE: MATHEMATICAL PHYSICS

STREAM: **SESSION V & VI**

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M**

DATE: **07/04/2009**

INSTRUCTIONS

This examination paper consists of **five** questions. Question 1 carries 40 marks while each of the other questions carries 15 marks. Answer **question 1** and any other **two questions**.

You may need the following **constants**.
Acceleration due to gravity, $g = 10m/s^2$

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Question 1 (40marks)

a) (i) Define the term, 'the scalar product' of two vectors $\overline{A}$ and $\overline{B}$. (2mks)
(ii) Hence state one physical application of the scalar product. (1mk)

b) The instantaneous displacement y of a progressive wave with speed v and frequency υ can be expressed as $y = A\exp\left[2\pi i\upsilon\left(\frac{x}{v} - t\right)\right]$, where A is the amplitude, x is the distance from the source and i is the complex notation. Determine the acceleration of the wave at t = 0. (3mks)

c) Evaluate the Fourier sine transform of e^{-x}. (3mks)

d) Given that $z_1 = 2 + 3i$ and $z_2 = 5 - 4i$, determine;
(i) $(z_1 z_2)^*$ (ii) the modulus of z_2 (iii) the argument of z_1 (5mks)

e) A force field is given by $\overline{F} = -3x^2\hat{x} + y^3\hat{y} + 5yz\hat{z}$. Determine the work done in the field in moving a particle along the paths $x = t^2+1$, $y = 2t^2$, $z = t^2$ from t = 0 to t = 2. (5mks)

f) (i) Explain the term, 'Hilbert Space.' (2mks)
(ii) State two properties of Hilbert Space. (2mks)

g) Determine the Laplace transform of $f(x) = 5Sin2x + 2Cos3x$. (2mks)

h) Two vectors are defined by $\overline{A} = 3\hat{x} + 2\hat{y} - 2\hat{z}$ and $\overline{B} = 2\hat{x} + 3\hat{y} + 2\hat{z}$ respectively.
Determine;
(i) a vector that is perpendicular to the plane containing vectors $\overline{A}$ and $\overline{B}$. (3mks)
(ii) the angle between the two vectors. (3mks)

i) The acceleration of a particle at time t is given by $\mathbf{a} = 6Cos3t\hat{x} + 9t^2\hat{y} - 4Sin2t\hat{z}$.
Determine the velocity v of the particle at any time t if the initial velocity was 4m/s. (4mks)

j) (i) Define the term, 'the Lagrangian' of a non-relativistic system. (1mk)
(ii) Write down the Euler-Lagrange equation, stating the meaning of symbols used. (2mks)

k) State Cauchy's Integral theorem (2mks)

Question 2 (15 marks)

a) State Green's theorem in a plane. (2mks)

b) Verify Green's theorem in the plane for $\oint_C \left[\left(xy + y^2\right)dx + x^2 dy\right]$ where C is the closed curve of the region bounded by y = x and y = x^2. (7mks)

c) Evaluate the line integral $\oint_C \mathbf{A.dr}$ where $\mathbf{A} = (3x - 2y)\hat{x} + x^2 z\hat{y} + y^2(z+1)\hat{z}$ for a plane rectangular area with vertices at (0,0), (1,0), (1,2), (2,0) in the x-y plane. (6mks)

Question 3 (15 marks)

a) (i) Define the term, a differential equation.' (1mk)
(ii) Hence distinguish between ordinary differential equations and partial differential equations. (2mks)

b) The rate of decay of a radioactive material is proportional to the amount of material present at any instant t. A certain radioactive material which initially had 100gm was found to have lost 20% of its original mass after 2 hours. Derive the specific relationship for the amount N of the material remaining at any instant t. (6mks)

b) Hermite polynomials are generated by the expression;

$$H_n(x) = \sum_{r=0}^{s} (-1)^r \frac{n}{(n-2r)r} 2x^{n-2r}$$

where n is a positive integer and $s = \frac{n}{2}$ for n even and $\left(\frac{n-1}{2}\right)$for n odd. Determine the first three Hermite polynomials. (6mks)

Question 4 (15 marks)

a) (i) Distinguish between an Eigen vector and an Eigen value of a linear transformation. (2mks)
(ii) Describe briefly the meaning of the term,' Dirac Notation.' (2mks)

b) An operator $\hat{A}$ is represented by the matrix $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$. Determine;

(i) the characteristic equation of $\hat{A}$. (3mks)
(ii) the eigen values of $\hat{A}$. (3mks)

c) The density d of a cube varies according to the expression $d = 2x^3 + y^2 + 3z^2$ where $0 \le x \le 2$, $0 \le y \le 2$ and $0 \le z \le 2$.Determine the total mass of the cube. (5mks)

Question 5 (15 marks)

a) Define the term, 'an irrotational vector' giving an example. (2mks)

b) An object of mass m falling freely in air experiences a viscous force given by $-kv$ where v is the velocity of the object and k is a constant.

(i) State two other forces acting on the object (2mks)

(ii) Formulate the equation of motion of the object (3mks)

(iii) Determine the general solution for the equation of motion of the object (6mks)

(iv) Given that velocity v of the object was zero initially, m = 0.5kg and k = 4N/m, determine the velocity of the object at time t = 2s. (2mks)