KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2010/2011 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE

COURSE CODE: **MATH 312**

COURSE TITLE: **ORDINARY DIFFERENTIAL EQUATIONS**

STREAM: **SESSION VI & VII**

DAY: **TUESDAY**

TIME: **9.00 – 11.00 A.M.**

DATE: **12/04/2011**

INSTRUCTIONS:

1. Question **ONE** is compulsory.
2. Attempt question ONE and any other TWO

PLEASE TURN OVER

QUESTION ONE: 30 MARKS

a. Find the particular solution to $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ given that $y = 1$ when $x = 1$ [4 marks]

b. Show that the given equation is an exact equation and hence find the general solution.

$$\frac{t^2}{x}\frac{dx}{dt} + 2t\ln x = 3\cos t$$ [6 marks]

c. Find the general solution of

$$\left(1+x^2\right)\frac{d^2y}{dx^2} = 2x\frac{dy}{dx}$$ [6 marks]

d. Define an auxillary quadratcequation and hence solve

$$2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 3y = 0$$ [4 marks]

e. Determine the solution of the following non-homogeneous equation by the method of variation of parameter

$$\frac{d^2y}{dx^2} + y = \tan x \sec x$$ [6 marks]

f. Solve $(D-2)^2 y = 8\left(e^{2x} + \sin 2x\right)$ [4 marks]

QUESTION TWO: 20 MARKS

a. Certain ODE are not exact but can be made exact y multiplying with an intergrating factor. Derive an expression for the factor. [10 marks]

b. Mr. Africaanmann retired last year at an age of 65 years. His initial retirement account has a principal of 9,000 (Ksh. ‘000), which was invested with a guaranteed interest rate of 5.25%

compounded continously. His badget calls for annual expenses of Ksh.200, 000 with projected inflation rate of 2.5%. Calculate

i. The balance in the account t years after his retirement.

ii. The time taken to use the entire amount in the account.

[10 marks]

QUESTION THREE: 20 MARKS

a) The following equation is related to biophysical limitations in the study of deep diving

$$y' = AY + B + Be^{-ax}$$

where a, b, A and B are constants show that the general solution of this equation is given by

$$y' = -\frac{B}{A} - \frac{b}{a+A}e^{-ax} + ce^{Ax}$$

Where c is an arbitrary constant. [8 marks]

b) The population of a constituency in 1994 and 2000 was 120,000 and 180,000 respectively. Find the year wen the population was $5^1/_3$ ten thousands, if the rate of growth is proportional to the population. [8 marks]

c) Find the particular intergral given that

$$(D^3 + 1)y = \cos(2x - 1)$$ [6 marks]

QUESTION FOUR: 20 MARKS

a) Find the particular solution to the equation given that

when $x = 0$, $y = 5$ and $\frac{dy}{dx} = 23$

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 15y = 0$$ [10 marks]

b) Show that the general solution to an ordinary differential equation whose auxilary equation has complex roots $p \pm iq$ is given by $y = e^{px}\left(A\cos qx + B\sin qx\right)$ where A and B are constants. [10 marks]

QUESTION FIVE: 20 MARKS

a) By using suitable transformation to reduce the equation to a separable equation, solve

$$\left(2x^3 + y^3\right)dx - 3xy^2 dy = 0$$ [8 marks]

b) Eliminate the constants to obtain the general equation whose general solution is

$y = c_1x^2 + c_2e^{2x}$ where c_1 and c_2 are arbitrary constants. [6 marks]

c) By using a suitable integrating factor solve

$\left(3x^4 y - 1\right)dx + x^5 dy = 0$ when $x = 1$, $y = 1$. [6 marks]