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University Examinations 2012/2013

THIRDYEAR, FIRST SEMESTER EXAMINATION FOR BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE,BACHELOR OF SCIENCE IN STATISTICS AND BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMA 2332/STA 2302: PROBABILITY AND STATISTICS IV

DATE: AUGUST 2013 **TIME: 2HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE – (30 MARKS)

(a) Let $(x_1, x_2 \dots, x_p)$ be a p- variate normal with density function given as

$f(\underline{x}) = ke^{1/2Q}$, where $Q = (\underline{x} - \mu)' \Sigma^{-1} (\underline{x} - \underline{\mu})$

k is a constant to be determined

$\mu = \mu_1, \mu_2, \dots \mu_p$

Σ is the covariance matrix

Determine the

(i) Value of k so that $f(\underline{x})$ is a pdf (6 Marks)

(ii) Moment generating functions of $f(\underline{x})$ (7 Marks)

(b) State and prove the generalized form of Chebychev's inequality. (7 Marks)

(c) Let three random variables x, y and z have joint p.d.f.

$$f(x, y, z) = \begin{cases} & 0 < x < 1 \\ kxyz^2, & 0 < y < 1 \\ & 0 < z < 3 \end{cases}$$

(i) Show that the constant $k = 4/9$ (5 Marks)

(ii) Find the conditional distribution of x given $y = {}^{1}/_{2}$ and $z = 1$ (5 Marks)

QUESTION TWO – (20MARKS)

(a) Use the non-generalized form of Chebycher's inequality to determine the number of times a coin must be tossed so that the probability that the ratio of the number of heads to the number of tosses will be between o.4 and 0.6 is at least 0.9. (6 Marks)

(b) Let x be normally distributed with mean μ and variance δ^2. Obtain the
   (i) Probability generating function of x. (4 Marks)
   (ii) Mean and variance from the probability generating function. (4 Marks)

(c) Suppose that x_1, x_2 and x_3 are independent random variables with unit variance. Let
$y_1 = x_1 + x_2 + x_3,\ \ y_2 = x_1 - x_2,\ \ y_3 = x_1 - x_3$
Find the covariance matrix for $\underline{y}$ where $\underline{y} = (y_1, y_2, y_3)$. (6 Marks)

QUESTION THREE – (20 MARKS)

(a) Let $\underline{x}^T = (x_1, x_2, x_3)$ have multivariate normal density given by
$$f(\underline{x}) = C\ exp\left\{-\frac{1}{2}(3x_1^2 + x_2^2 + 5x_3^2 - x_1x_2 - 3x_2x_3)\right\}$$

Determine
   (i) Covariance matrix Σ (5 Marks)
   (ii) Marginal density of x_1x_3 (5 Marks)
   (iii) Conditional density of x_1 given x_2x_3 . (4 Marks)

(b) (i) State the weak law of large numbers. (3 Marks)
   (ii) Prove the weak law of large numbers. (6 Marks)

QUESTION FOUR – (20 MARKS)

(a) Let $\underline{x}^T = (x_1, x_2)$ be a random vector P.G.F
$$P = (s_1 s_2) = (q_1 + p_1 s_1)^{n_1} + (q_2 + p_2 s_2)^{n_2} + (q_1 + q_2 + p_1 p_2 s_1 s_2)^{n_1 + n_2}$$

Find the
   (i) Mean of $x_1\ and\ x_2$ (6 Marks)
   (ii) Variance of x_1 (4 Marks)
   (iii) Covariance if $x_1\ and\ x_2$ (4 Marks)

(b) Let x and y have a joint p.d.f. given by
$$f(x,y)\begin{cases} k(1-y) 0 \le x \le y \le 1 \\ 0, elsewhere \end{cases}$$
   (i) Calculate the value of k. (3 Marks)
   (ii) Calculate the $prob\left(x \le {}^{3}/_{4}, y \ge {}^{1}/_{2}\right)$ (4 Marks)

QUESTION FIVE – (20 MARKS)

(a) State and probe the central limit theorem (8 marks)

(b) Let x and y have the following distribution

$$f(x_1 x_2) = \frac{1}{2\Pi\sqrt{1-p^2}} e^{-\frac{1}{2(1-p^2)}\left[x_1^2 + 2px_1x_2 + x_3^2\right]}$$

Show that $f(x_{1,} x_2)$ is a bivariate density function. (8 Marks)

(c) Discuss the importance of the central limit theorem in statistics. (4 Marks)