EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2014/2015

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 208: ORDINARY DIFFERENTIAL EQUATIONS 1

DATE: DECEMBER 17, 2014 **TIME: 08:00 – 10:00AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) State the order , degree and linearity of the differential equation

$$\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}=\left(\frac{d^2y}{dx^2}\right)^3$$ (4 marks)

b) Obtain a differential equation associated with the primitive below

$$y = Ae^{3x} + Bxe^{3x}$$ (4 marks)

c) Show that the differential equation is exact. Hence, solve it.

$$(3x^2 + y + 1)dx + (3y^2 + x + 1)dy = 0$$ (4 marks)

d) Demonstrate that the equation is variable separable and hence solve it.

$$(x+1)\frac{dy}{dx} = x(y^2+1)$$ (4 marks)

e) Find the general solution of

$$x\frac{dy}{dx} + y = x\sin x$$ (4 marks)

f) If you are using method of undetermined coefficients to solve the equation below, what is your best guess for the form of the particular solution?

$$\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 9y = e^{-3t}$$ (3 marks)

g) Find the general solution of the following differential equation

$$\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 25y = 0$$ (3 marks)

h) A cup of coffee at $190^\circ F$ is left in a room temperature of $70^\circ F$. At time $t = 0$, the coffee is cooling at $15^\circ F$ per minute. Assuming that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings. Find the function that models the cooling of the coffee.

(4 marks)

QUESTION TWO

a) Show that the differential equation

$$(3xy - 2y^2)dx + (x^2 - 2xy)dy = 0$$

has an integrating factor which is a function of x alone, hence solve the differential equation.

(7 marks)

b) Solve the differential equation

$$dx + (1 - x^2)\cot y dy = 0$$ (7 marks)

c) If the population of a country doubles in 50 *years*, in how many years will it treble under the assumption that the rate of increase is proportional to the number of inhabitants.

(6 marks)

QUESTION THREE

a) Find the solution of the following system of equations

$$\frac{dx}{dt} + \omega y = 0$$
$$\frac{dy}{dt} - \omega x = 0$$ (6 marks)

b) Show that the equation

$$\{1 + (x + y)\tan y\}dy + dx = 0$$

has an integrating factor of the form $(x + y)^n$ where n is a constant, hence solve the equation. (7 marks)

c) Transform the Bernoulli equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

into first degree, first order and hence solve the equation

$$x\frac{dy}{dx} + y = y^2x^2$$ (7 marks)

QUESTION FOUR

a) Use the method of undetermined coefficients to find the general solution of the differential equation.

$$\frac{d^2y}{dx^2} + 9y = 6\sin 3x$$ (10 marks)

b) By the use of the method of variation of parameters, obtain the general solution of the differential equation

$$\frac{d^2y}{dx^2} + y = \tan x$$ (10 marks)

QUESTION FIVE

a) Find the general solution of the following differential equation

$$\frac{dy}{dx} = \frac{-3x - y - 2}{x + y + 2}$$ (8 marks)

b) Solve the following differential equation

$$xyp^2 + (x^2 + xy + y^2)p + x^2 + xy = 0$$

Where $p = \dfrac{dy}{dx}$ (5 marks)

c) Find a power series solution to the differential equation

$$y'' + y = 0$$

(7 marks)

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