KISII UNIVERSITY

MATH 318: CALCULUS III **TIME: 2 HOURS**

AUGUST 2017 EXAMINATION

INSTRUCTIONS: *Answer question ONE (compulsory) and any other TWO questions.*

QUESTION ONE (30MARKS)

(a) Find C for Lagrange's mean value theorem for the function $f(x) = x^2 + 2x - 1$ for the interval $[0 \leq x \leq 1]$. (3mks)

(b) Discuss the nature of the following infinite series:

(i) $1 + \frac{6}{4} + \frac{8}{4} + \frac{10}{4} + - - - - - - - \infty$ (3mks)

(ii) $3 + \frac{3}{2} + \frac{3}{2^2} + \frac{3}{2^3} + - - - - - - - \infty$ (3mks)

(c) Evaluate the following improper integrals without using tables:

(i) $\int_0^\infty \frac{dx}{x^2+1}$ (4mks)

(ii) $\int_0^1 \frac{dx}{\sqrt{1-x}}$ (4mks)

(d) Find the laplace transformation of $F(t) = a$ where a is a constant. (3mks)

(e) Evaluate the surface integral $\iint_S (\nabla x F).nds$ by transforming into a line integral, S being that part of the surface of the paraboloid $Z = 1 - x^2 + y^2$, $Z \geq 0$ and $F = yi + zj + xk$.

(6mks)

(f) By transforming the integral to polar coordinates, evaluate $I = \iint_R [1 - \sqrt{x^2 + y^2}\ ]\,dxdy$ where R is the region bounded by the circle $x^2 + y^2 = 1$. (4mks)

QUESTION TWO (20MARKS)

(a) (i) State Stoke's theorem. (2mks)

(ii) Verify Stoke's theorem for the function $F = x^2 i + xyj$ taken round the rectangle $x = 0, y = 0, x = a, y = b$ on the plane $z = 0$. (10mks)

(b) Use D'Alembert's ratio test to investigate the convergence of a series whose nth term is $\frac{n^2}{2^n}$

(3mks)

(c) Show that $\int_0^{\ln 2} x^{-2} e^{-\frac{1}{x}} dx$ is an improper integral. Hence test the convergence of the integral. (5mks)

QUESTION THREE (20MARKS)

(a) Define the following:

(i) a periodic function. (1mk)

(ii) an odd function. (1mk)

(iii) an even function. (1mk)

(b) Show that $f(x) = x^2 + \cos x + 8$ is an even function. (3mks)

(c) Use the new theorem for extension of integration by parts to find $\int x^3 \cos x \, dx$ (3mks)

(d) (i) Expand $f(x) = x^2, -\pi < x < \pi$ in a Fourier series. (8mks)

(ii) Using your result prove that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + - - - - - = \frac{\pi^2}{12}$ (3mks)

QUESTION FOUR (20MARKS)

(a) (i) State Greens theorem. (2mks)

(ii) Using Greens theorem evaluate $\oint_c \{y(2xy-1)dx + x(2xy+1)dy\}$ where C is the circle $x^2 + y^2 = 9$ (7mks)

(b) Prove that for every convergent series $\sum u_n$, $\lim_{n\to\infty} u_n = 0$ (4mks)

(c) Test for convergence of the following series using Cauchy's fundamental test:

(i) $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1}$ (4mks)

(ii) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ (3mks)

QUESTION FIVE (20MARKS)

(a) Evaluate $\int_c (xydx + xy^2dy)$ by transforming into a surface integral where c is the square on the xy-plane with vertices $(1,0), (-1,0), (0,1), (0,-1)$. (8mks)

(b) Evaluate $\int x^4 e^{2x} dx$ (4mks)

(c) Let $I = \int_{x=0}^{5} \int_{y=0}^{5-x} dxdy$

(i) Sketch the region of integration R. (2mks)

(ii) Evaluate the integral I. (3mks)

(d) Evaluate the double integral $\iint_R (6x^2 + 3y^2 + 2)dxdy$ where R is the region bounded by the lines $0 \le x \le 1$, $0 \le y \le 2$. (3mks)