

**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

 **KISII LEARNING CENTRE**

**FIRST YEAR SEMESTER TWO EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)**

**SMA 103- LINEAR ALGEBRA I TIME- 2HOURS**

**INSTRUCTIONS: Answer question one (compulsory) and any other two questions**

**QUESTION ONE (30 MARKS)**

1. Consider the following systems of linear equations;

 *x +2y = 4*

 *2x – y = 3*

 *3x + y = k*

Find the value of *k* for which the system is consistent (3mks)

1. Use Gauss- Jordan Elimination method to solve the following systems of linear equations (6mks)

*x1 + x2 +2x3 = 8*

*x1 + 2x2 -3x3 = -1*

*3x1 -7 x2 +4x3 = 10*

1. Find the values of **λ** for which the determinant of the matrix below is equal to zero

 $\left[\begin{matrix}λ+1&0&0\\4&λ&3\\2&8&λ+5\end{matrix}\right]$ (5mks)

1. Let A = $\left[\begin{matrix}1&2&-2\\2&-3&10\\0&1&-3\end{matrix}\right]$ , find the adjoint of *A* and hence determine the value of A-1 . (8mks)

1. i) State the Cramer’s rule (2mks)

ii) Use Cramer’s rule to solve the following systems of linear equations;

 - *x1 + 2x2 -3x3 = 3*

*2x1 +x3 = 0*

*3x1 -4 x2 +4x3 = 6* (6mks)

 **QUESTION TWO ( 20 MARKS )**

1. i) Define the term Basis of a vector space *V*  (2mks)

ii) Let *S = { V1=(1,2,1), V2 =(2,9,0), and V3 =(3,3,4)}.* Show that the set S is a basis

 for $R$3  (5mks)

1. Show that $<$*u, v> = u1v1 +2u2v2* where u = *(u1, u2), v = (v1, v2)* is an inner product on $R$2 (6mks)
2. i) Define the term Linear transformation (1mk)

ii) Show that T:$ R$3→$ R$2 defined by T**=**$\left[\begin{matrix}x\_{1}\\x\_{2}\\x\_{3}\end{matrix}\right]=\left[\begin{matrix}x\_{1}+x\_{2}\\x\_{2}-x\_{1}\end{matrix}\right]$is a linear transformation(6mks)

 **QUESTION THREE (20 MARKS)**

a) i) Define the term a symmetric matrix (1mk)

ii) Prove that a symmetric matrix of order 2 is diagonalizable (4mks)

iii) State the Cayley – Hamilton’s theorem and use it to verify for the matrix

 A=$\left[\begin{matrix}1&-3\\2&5\end{matrix}\right]$(4mks)

b) i) Show that the determinant of a second order matrix with identical rows is zero

(2mks)

ii) Consider the matrices A=$\left[\begin{matrix}2&-1\\4&3\end{matrix}\right]$ , B=$\left[\begin{matrix}1&-4\\4&-1\end{matrix}\right]$ determine whether these matrices

 Commute and hence find the commutator (4mks)

1. Use Cramer’s rule to find the point of intersection of the three planes defined by;

 *x+ 2y -z = 4*

*2x-2y+3z = 3*

*4x+3y-2z = 5* (5mks)

 **QUESTION FOUR (20 MARKS)**

1. Calculate the area of the triangle whose vertices are A(1,0), B(2,2) and C(4,3) by use of the method of determinants (4mks)
2. Consider the vectors *u= (1,-3, 7)* and *v= (8,-2,-2).* Find *u.v* and the angle between them (5mks)
3. i) Let *V=*$R$*3* with standard operations and S={ (1,2,3 ), (0,1,2), (-2,0,1 )}≤$ R$*3*. Does S Span V? (3mks)

ii) Let *V=*$R$*3* and S={ (-4,-3,4 ), (1,-2,3), (6,0,0 )}.Determine whether S is linearly independent. (4mks)

1. Find the basis and dimension for the solution space of the homogeneous system

 *2x1 + x2 +3x3 = 0*

*x1 +5x3 = 0*

*x2+x3 = 0* (4mks)

**QUESTION FIVE (20 MARKS)**

1. Apply the Gram-Schmidt process to construct an orthonormal basis set for *B={(1,1,0), (1,2,0), (0,1,2)}* of $R$*3*. (7mks)
2. Show that the transformation *T(x)=2x+1* is not a linear transformation (3mks)
3. Find the eigen values of the matrix A=$\left[\begin{matrix}5&2\\9&2\end{matrix}\right]$ (5mks)
4. Let A=$\left[\begin{matrix}1&2\\0&1\end{matrix}\right]$ . Determine whether or not A is diagonalisable (5mks)