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MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

CITY CAMPUS

MAS 809: EPIDEMIC MODELING

Date: 20th February, 2015 Time: 9.00 - 12.00 noon

INSTRUCTIONS:

- Answer ANY THREE questions.

Question 1 (20 Marks)

a) Define the term "epidemic" **(2 Marks)**

b) Consider the S – I Model defined by the equations

$$\frac{dS(t)}{dt} = -c\lambda(t)S(t),$$

$$\frac{dI(t)}{dt} = c\lambda(t)S(t) - \upsilon I(t),$$

where $\lambda(t) = \beta\frac{I(t)}{N(t)}, N(t) = I(t) + S(t)$ and c and υ are some constants. Show that if $S(t) \cong N(t)$ then

$$I(t) = I_0 e^{[(\beta c - \upsilon)t]} \dots\dots\dots\dots\dots (1)$$

where $I_0 = I(0)$ and that the time taken if there is an epidemic for the number of infected to double is

$$t_d = \frac{ln2}{(\beta c - \upsilon)} \dots\dots\dots\dots\dots (2)$$

(5 Marks)

c) Using the results in Equation ....(1) or otherwise, determine the conditions under which an epidemic will arise **(3 Marks)**

d) Consider the epidemic chain binomial model. Let $i_t : t = 0, 1, \cdots, r\ (i_{r+1} = 0)$denote the number of infectives at time t since the onset of the epidemic

Question 2 (20 Marks)

Consider the following data on observed and expected frequencies for the outbreak of size 5:

	Number of Cases					
	$O_1(E_1)$	$O_2(E_2)$	$O_3(E_3)$	$O_4(E_4)$	$O_5(E_5)$	Total
Overcrowded	112(115.3)	35(35.7)	17(16.4)	11(9.8)	6(3.8)	181
Crowded	115(153.5)	41(47.5)	24(21.8)	15(13.1)	6(5.1)	241
Uncrowded	156(154.2)	55(47.7)	19(21.9)	10(13.1)	2(5.1)	242
Total	423	131	60	36	14	664

a) Show that in testing the hypothesis:

   H_0: *Number of cases is independent of level of crowding*

   versus

   H_1: *Number of cases increases with increased level of crowding,*

   H_0 is not rejected.

b) Given the results in 2(a), calculate the estimate of the expected number of cases, including the introductory case, in households of size five affected through one primary case. Determine the standard error of the estimate.

(20 Marks)

Question 3 (20 Marks)

Complete the following table by determining the appropriate chain binomial probabilities for each given chain. Use the notation in 3(b) and assume one introductory case.

	Household size		
Chain	$5(S_0 = 4)$	$4(S_0 = 3)$	$3(S_0 = 2)$
1			

$1 \to 1$ $1 \to 1 \to 1$ $1 \to 2$ $1 \to 1 \to 1 \to 1$ $1 \to 1 \to 2$ $1 \to 3$ $1 \to 1 \to 1 \to 2$ $1 \to 1 \to 2 \to 1$			

b) Let $\beta_j = p\begin{pmatrix} j \text{ of the susceptibles of the household} \\ \text{become cases by the end of the outbreak} \end{pmatrix}$. Determine using the results in 3(a):

i) $\theta_1, \theta_2, \theta_3$ and θ_4 with respect to household of size 4 ($S_0 = 4$)

ii) θ_1, θ_2 and θ_3 with respect to household of size 3 ($S_0 = 3$) (20 Marks)

Question 4 (20 Marks)

Consider the "general epidemic" model with susceptibles $S(t)$, infectives $I(t)$ and immunes $R(t)$. Assuming homogeneous mixing among the classes and with continuous time t, let

$$\frac{dS(t)}{dt} = -\beta S(t)I(t),$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \alpha I(t),$$

$$\frac{dR(t)}{dt} = \alpha I(t)$$

with initial conditions $(S(0),\ I(0),\ R(0)) = (S_0, I_0, 0)$ and α and β are some constant numbers. Assume $N = S(t) + I(t) + R(t)$N = S(t) + I(t) + R(t)

a) Explain the formulation with respect to disease spread dynamics.

(3 Marks)

b) Show that:

i) $\frac{dI(t)}{dt} = \beta I(t)(S_0 - \rho)$, where $\rho = \frac{\alpha}{\beta}$ **(3 Marks)**

ii) $S(t) = S_0 e^{\frac{R(t)}{\rho}}$ **(4 Marks)**

iii) for $N(t)$ such that $N(t) = \rho - \upsilon$ where $\upsilon \ll \rho$ with $N(t) \approx S_0$, $R(\infty) \approx 2\upsilon$ and $S(\infty) \approx \rho - \upsilon$ **(8 Marks)**

c) Comment on the results of 4(b)(iii). **(2 Marks)**

Question 5 (20 Marks)

a) State the assumptions made in the chain binomial model and prove that for an epidemic chain $i_0 \to i_1 \to \cdots \to i_r$,

$$p(i_0 \to i_1 \to \cdots \to i_r) = \frac{s_0!}{i_1!\, i_2! \cdots i_r!\, s_r!} \prod_{t=1}^{r} p_{i_t}^{i_{t+1}} q_{i_t}^{s_{t+1}}$$

(6 Marks)

b) Consider a household of size 4 with two introductory cases

i) List all the possible epidemic chains **(4 Marks)**

ii) Determine the probability of each chain listed in b(i) **(4 Marks)**

c) Let $\theta_j = p\begin{pmatrix} \text{the number of infected is } j \text{ at the end of the} \\ \text{infectious process, for the household} \end{pmatrix}, j =$ 0, 1, 2, 3, 4. Determine $\theta_1, \theta_2, \theta_3$ and θ_4 in terms of the probabilities obtained in b(ii) **(6 Marks)**