MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

CITY CAMPUS

MAS 818: TIME SERIES ANALYSIS

Date: 21st February, 2015 Time: 9.00 - 12.00 noon

INSTRUCTIONS:

- Answer ANY THREE questions.

MASTER OF SCIENCE IN APPLIED STATISTICS

MAS 818 TIME SERIES ANALYSIS

ANSWER ANY THREE QUESTIONS

1. A)Show that the $AR(2)$ series will exhibit exponential decay as $\mu \rightarrow \infty$ (Marks 10)

   B)Show that the autocovariance function of an $AR(p)$ series yields a linear homogenous difference equation and state its general solution (Marks 10)

2. A) Derive the autocorrelation function of the $ARMA(1,1)$ series (Marks 10)

   B) Consider the k^{th} order autoregressive prediction of X_{k+1}
   i.e $\hat{X}_{k+1} = \phi_{k_1}X_k + \cdots + \phi_{kk}X_1$
   obtained by minimizing $E(X_{k+1} - \hat{X}_{k+1})^2$. Show that the $K\ th$ partial autocorrelation value is given by $\phi(k) = \phi_{kk}$ (Marks 10)

3. Suppose (X_t) is a stationary and invertible $ARMA\ (p,q)$ model
   A) Derive an l-step ahead forecast function $\{X_{t+l}\}$ from the origin t and the corresponding l-step ahead forecast error $\{\varepsilon_t(t)\}$. (7 marks)
   B) Show that the minimum mean square forecast function of X_{t+l} is given by the conditional mean
   $$X_t(l) = E(X_{t+l}/Ft)$$
   Where Ft is the history of the process. (Marks 14)

   C) For the $ARMA(1,1)$ process given by the equation $X_t - 0.6X_t = Z_t + 0.3Z_{t-1}$, $Z_t \sim N(0, \sigma^2)$ find the stationary and invertible solutions. Hence or otherwise find $Var(X_t)$ (7 marks)

4. A) Evaluate the spectral density function of an $AR(2)$ process given by
   $X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} e_t$ (7marks)
   B) Let $X_t = a + bt + ct^2 + e_t$, where a, b and c are constants use the differencing method to remove the quadratic tend (6marks)
   C) Evaluate ΔX_t in $X_t = e^{i\lambda t}$ by differencing, hence describe the changes in amplitude and phase angle. (7marks)

5. A) Derive the Box- Jenkins general forecasting methodology generated by a stationary autoregressive moving average $(ARMA)$ process. (10marks)

   B) Transform a moving average filter $\{X_t\}$ into another series $\{Y_t\}$ by the linear operator given that

   $X_t = e^{i\lambda t}$ and $Y_t = \sum_{j=-\infty}^{\infty} a_j X_{t-j}$

   Where

   $$a_j = \begin{cases} \frac{1}{2m+1}, & j = 0, \mp1, \mp2, \ldots\ldots, \mp m \\ 0, & otherwise \end{cases}$$ [10marks]