MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2013/2014

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE

(MAIN CAMPUS)

MMA 805: GENERAL TOPOLOGY I

Date: 11th December, 2013 *Time: 9.00 a.m. - 12.00 noon*

INSTRUCTIONS:

- Answer Question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.

Question One (30 marks):

1. (a) Outline the procedure for showing that a point p in a topological space (X, τ) is a limit point of a subset $A \subset X$.

(3 marks)

(b) Let X be an arbitrary infinite set. Define the class $\mathcal{B}$ of subsets of X by

$$\mathcal{B} := \{O \subset X : X \setminus O \text{ is finite}\} \cup \{\emptyset, X\}$$

Show that $\mathcal{B}$ is a topology in X. **(3 marks)**

(c) Let X := {a, b, c, d, e} and consider the topology

$$\tau := \{\{a, b\}, \{b, c\}, \{b\}, \{a, b, c\}, \emptyset, X\}.$$

further, let $B := \{a, c\}$. Compute the boundary of B. **(3 marks)**

(d) Explain what is meant by a property of a family of subsets of a topological space (X, τ) being topological. Give an example of properties which are not topological. **(3 marks)**

(e) Distinguish between identification topology and relative topology.

(6 marks)

(f) Show that a regular space need not be T_1-space.

(3 marks)

Question Two (20 marks):

1. (a) Give an example to show that an open map is not necessarily continuous. **(6 marks)**

(b) Let (X, τ_X), and (Y, τ_Y) be topological spaces. Suppose $f : X \mapsto Y$ is a map with property:

for each point $p \in X$ and any neighbourhood N_p of p there corresponds a neighbourhood $N_{f(p)}$ of $f(p)$ such that $f(p) \in N_{f(p)} \subset f(N_p)$.

Show that f is open **(8 marks)**

(c) Define what is meant by a homeomorphism.

Let $f : X \mapsto Y$ be a bijective map which has the property:

$$f(\overline{A}) = \overline{f(A)} \quad \forall A \subset X$$

Show that f is a homeomorphism. **(6 marks)**

Question Three (20 marks):

1. (a) Let X := [-1,1] $\subset \mathbf{R}$ and (X,τ) be a topological space with τ being relative topology on X with respect to the usual topology in $\mathbf{R}$. Further let $f : X \mapsto \{0,1\}$ be the characteristic map on $[\frac{1}{2}, 1] \subset X$. Show that the identification topology on {0,1} induced by f coincides with the Sierpinsky's topology in {0,1}.

   (5 marks)

   (b) Let (X, τ_X) and (Y,τ_Y) be topological spaces and $f : X \mapsto Y$ be an identification. Show that f is open iff $f^{-1}(f(B))$ is open for each open $B \subset X$ **(8 marks)**

   (c) Give an example to show that a Housdorff space is not necessarily regular. **(7 marks)**

   (d)

Question Four (20 marks):

1. Let X be a Housedorf space. Show that:

   (a) Each finite subset of X is closed. **(6 marks)**

   (b) For each $A \subset X$, a point $p \in X$ is a limit point of A iff each neighbourhood of p contains infinitely many points of A.

   (14 marks)

Question Five (20 marks):

1. (a) Let X be an arbitrary set. Suppose the class $\mathcal{B}$ of subsets of X has the properties

   i. X = $\cup$\{B:B$\in \mathcal{B}$\}.

   ii. $\forall$ B_1, $B_2 \in \mathcal{B}$, $B_1 \cap B_2$ is expressible as a union of members of $\mathcal{B}$

   (a) Show that $\mathcal{B}$ is a base for a topology on X. **(14 marks)**

   (b) Is the converse true? **(6 marks)**