MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2013/2014

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

(MAIN CAMPUS)

MMA 817: ORDINARY DIFFERENTIAL EQUATIONS I

Date: *16th December, 2013* ***Time:*** *9.00 – 12.00 noon*

INSTRUCTIONS:

- **Answer ANY THREE questions.**
- **Start each question on a fresh page.**
- **Indicate question numbers clearly at the top of each page.**
- **Observe further instructions on the answer booklet.**

Question One: (20 marks)

(a)

(i) Prove the following statement: The continuously differentiable functions $y_1(t), y_2(t) : (t_1, t_2) \to \mathbb{R}$ are linearly dependent iff $W_{y_1y_2)}(t) = 0$ holds for all $t \in (t_1, t_2)$

Find the Wronskian of the functions

(ii) $y_1(t) = \sin t$ and $y_2(t) = 2\sin t$

(iii) $y_1(t) = \sin t$ and $y_2(t) = t\sin t$

[7 marks]

(b) Find two linearly independent solutions of the equation

$$y'' + y' - 2y = 0$$

[4 marks]

(c) Prove that if $a_1, a_0 : (t_1, t_2) \to \mathbb{R}$ are continuous functions, and y_1, y_2 are twice continuously differentiable solutions of the equation

$$y'' + a_1(t)y' + a_0(t)y = 0$$

Then the Wronskian is a solution of the equation

$$W'_{y_1y_2}(t) + a_1(t)W_{y_1y_2}(t) = 0$$

Therefore, for any $t_0 \in (t_1, t_2)$, the Wronskian is given by the expression

$$W_{y_1y_2} = W_0 e^{A_1(t)}$$

where $W_0 = W_{y_1y_2}(t_0)$ and $A_1(t) = \int_{t_0}^{t} a_1(\tau)d\tau$ [9 marks]

Question Two: (20marks)

(a) Find the real-valued fundamental solutions of

$$\frac{d^2I}{dt^2} + 2\alpha\frac{dI}{dt} + \omega^2 I = 0$$

where

$$\alpha = \frac{R}{(2L)}$$

$$\omega^2 = \frac{I}{(LC)}$$

in cases (i) and (ii) below

Question Five (20 marks)

(a) Find the functions x_1, x_2 solutions of the first order, 2×2, constant coefficients, homogeneous differential system

$$x_1' = x_1 - x_2$$

$$x_2' = -x_1 + x_2$$

[12 marks]

(b) Express the second order equation

$$y'' + 2y' + 2y = \sin(at)$$

as a first order system **[8 marks]**

====================== END ======================

(i) Case (a): $R = 0$

(ii) Case (b): $R < \sqrt{4L/C}$

[12 marks]

(b) Find the solution to the initial value problem

$$9y'' + 6y' + y = 0, \quad y(0) = 1, \quad y'(0) = \frac{5}{3}$$

[8 marks]

Question Three: (20 marks)

(a) Find a second solution y_2 linearly independent to the solution $y_1(t) = t$ of the differential equation

$$t^2y'' + 2ty' - 2y = 0$$

[12 marks]

(b) Find all solutions to the inhomogeneous equation

$$y'' - 3y' - 4y = 3e^{2t}$$

[8 marks]

Question Four: (20marks)

(a) Define the following terms (i) linear differential equation (ii) Wronskian [4 marks]

(b) Find the general solution of the following inhomogeneous differential equation

$$y'' - 5y' + 6y = 2e^t$$

using the method of variation of paremeters [16marks]