# MASENO UNIVERSITY

## UNIVERSITY EXAMINATIONS 2013/2014

FIRST YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED MATHEMATICS

(MAIN CAMPUS)

## MMA 839: NUMERICAL ANALYSIS I

***Date:** 11th December, 2013* | ***Time:** 9.00 – 12.00 noon*

**INSTRUCTIONS:**

- **Answer ANY THREE questions.**
- **Start each question on a fresh page.**
- **Indicate question numbers clearly at the top of each page.**
- **Observe further instructions on the answer booklet.**

## Question One: (20 marks)

(a) Define the term truncation error. Write down Taylor series for a function f(x) expanded a round a general point $x_n$. Obtain a second degree polynomial that approximates

$$f(x) = \sqrt{1+x}, \quad x \in [0, 0.1]$$

using Taylor series expansion about $x = 0$. Use the result to approximate $f(0.05)$ and bound the truncation error **[10 marks]**

(b) What is an ill-conditioned system? Solve the system below by Gaussian elimination using 5 significant figures

$$x_1 + 5x_2 = 17$$

$$1.500x_1 + 7.501x_2 = 25.503$$

Hence improve accuracy in one step. **[10 marks]**

## Question Two: (20marks)

(a) Find the eigenvalues and an eigenvector corresponding to the least eigenvalue of the matrix A given by

$$A = \begin{bmatrix} 3 & 2 & 5 \\ 6 & -5 & 3 \\ -24 & 38 & 2 \end{bmatrix}$$

**[10 marks]**

(b) Determine the dominant and least eigenvalues of

$$A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$$

using iteration. Use 2 significant figures in your computation **[10 marks]**

## Question Three: (20 marks)

(a) Solve the system of linear equations

$$1.4x_1 + x_2 + 50x_3 = 56.2$$

$$27.1x_1 + 6x_2 - x_3 = 92.3$$

$$1.6x_1 + 12x_2 + x_3 = 29.8$$

by Gauss-Seidel iteration. When do you stop the iteration?

**[11 marks]**

(b) Solve the same system in (a) above by SOR method using $\omega = 1.01$

**[9 marks]**

## Question Four: (20marks)

(a) Use Lagrange interpolation to find the second degree polynomial passing through the points

| $x$ | 0 | 1 | 3 |
|---|---|---|---|
| $f(x)$ | 1 | 3 | 35 |

Hence determine the value of f(2). **[6 marks]**

(b) Prove the following relation

$$E = (1 - \nabla)^{-1}$$

and hence derive the Newton Backward Difference Formula. Using the formula obtain a polynomial for the following data

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | $-3$ | 1 | 11 | 33 | 73 | 137 | 231 |

**[14marks]**

## Question Five (20 marks)

(a) Given a $2 \times 2$ matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Find its eigenvalues and eigenvectors **[5 marks]**

**(b)** Use the power method to determine the spectral radius of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}$$

**[15 marks]**

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