MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2012/2013

SECOND YEAR SECOND SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF SCIENCE AND
BACHELOR OF EDUCATION (SCIENCE) WITH
INFORMATION TECHNOLOGY
(MAIN CAMPUS)

SPH 205: MATHEMATICAL METHODS FOR PHYSICS II

Date: 18th July, 2013 ***Time: 11.00 a.m. – 1.00 p.m.***

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UNIVERSITY EXAMINATIONS FOR 2012/2013 ACADEMIC YEAR

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SPH 205: MATHEMATICAL METHODS FOR PHYSICS II

TIME: 2 HOURS

INSTRUCTION

Answer question ***ONE*** and any other ***TWO*** questions in **section B.**

SECTION A

QUESTION ONE (30 MARKS)

a) (I) Given that $z = x^2y^2 + 3xy$, determine $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ **(2 marks)**

(II) Given a function $f = f(x_1, x_2, \ldots x_n)$, write an expression for the total differential **(1 mark)**

b) (I) Evaluate $\left(\frac{1+i\sqrt{3}}{1-i\sqrt{3}}\right)^{10}$ leaving your answer in the form $a + ib$. **(3 marks)**

(II) Show that $f(z) = z^2$ is analytic. **(2 marks)**

(III) Define the following terms.

(i) Power series **(1 mark)**

(ii) Essential singularity **(2 marks)**

(IV) State the fundamental theorem of algebra. **(1 mark)**

c) (I)Find the determinant of $\begin{pmatrix} 3 & 2 & 0 & 1 \\ 4 & 0 & 1 & 2 \\ 3 & 0 & 2 & 1 \\ 9 & 2 & 3 & 1 \end{pmatrix}$ **(4 marks)**

(II) Given the system of linear equations

$a_1x_1 + a_2x_2 = a_3$

$b_1x_1 + b_2x_2 = b_3$, obtain an expression for x_2 in form of a ratio of two determinants. **(3 marks)**

d) Define the following terms:

(I) Eigenvalue and eigenvector **(2 marks)**

(II) Hilbert space **(1 mark)**

e) Show that in a Fourier series of a function $f(x)$ in the interval $[-P \;\; P]$, the Fourier coefficient a_0 is given by

$$a_0 = \frac{1}{P}\int_{-P}^{P} f(x)dx$$ **(4 marks)**

f) (I) Define a first order linear Ordinary differential equation. **(1 mark)**

(II) Solve the equation $\frac{dy}{dx} + y = e^{x}$ **(3 marks)**

SECTION B

Answer ***ANY TWO*** questions in this section.

QUESTION TWO (20 MARKS)

a) Determine the partial second derivatives of $f(x,y) = e^{2x}\cos(y-x)$, hence show that the second partial derivative is a commutative operation. **(9 marks)**

b) Derive the De Moivre's theorem. **(5 marks)**

c) Find the cuberoot of 8. **(6 marks)**

QUESTION THREE (20 MARKS)

a) Derive the Cauchy Riemann Equations from first principles. **(9 marks)**

b) Expand $\frac{1}{(z+1)(z-3)}$ in a Laurent series about the point $Z = -1$ **(7 marks)**

c) Define the term ' determinant of a $2 \, x \, 2$ matrix', hence find the determinant of the matrix

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 1 & -3 \\ 4 & 0 & 1 \end{pmatrix}$$ **(4 marks)**

QUESTION FOUR (20 MARKS)

a) (I) Name an eigen value equation in quantum mechanics. **(1 mark)**

(II) Let $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$. Find the eigen values of A and the associated eigenvectors. **(12 marks)**

b) Show that the differential equation

$$(x^2 + y^2)dx + 2xydy = 0$$ is exact,

hence solve it. **(7 marks)**

QUESTION FIVE (20 MARKS)

a) Diagonalize the matrix $\begin{pmatrix} p & -q \\ q & p \end{pmatrix}$ where p and q are real numbers and $q \neq 0$ **(14 marks)**

b) Define a Fourier series of a function $f(x)$. **(2 marks)**

c) Given the box function which can represent a single pulse,

$f(x) = \begin{cases} 1, & -a \leq x \leq a \\ 0, & x > a \end{cases}$, find the Fourier transform of $f(x)$ **(4 marks)**

END