MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2012/2013

SECOND YEAR SECOND SEMESTER EXAMINATIONS
FOR THE DEGREE OF BACHELOR OF SCIENCE AND
BACHELOR OF EDUCATION (SCIENCE) WITH
INFORMATION TECHNOLOGY
(MAIN CAMPUS)

SPH 204: OSCILLATIONS AND WAVES

***Date:** 23rd July, 2013* ***Time:** 2.30 – 4.30 p.m.*

SPH 204: OSCILLATIONS AND WAVES

INSTRUCTION:

- ✓ Attempt **Question 1** in section **1** and **ANY two** questions from section **2.**

SECTION 1: COMPULSORY

QUESTION 1: 30MRKS

a) Define the following terms. **(2 mrks)**

A. Damped oscillations

B. Acoustics

C. Resonance

D. Elastic wave

b) Distinguish between transverse and longitudinal waves. **(2 mrks)**

c) State the difference between phase velocity and group velocity of a wave and give their relationship. When are the two velocities equal? **(3 mrks)**

d) At a certain point in the oscillation of a block + spring system, the kinetic energy is $\mathbf{0.1\,J}$ and the potential energy is $\mathbf{0.3\,J}$. The amplitude is $\mathbf{20cm}$ and the period is $\mathbf{0.8s}$. Find the mass of the block and the spring constant. **(2 mrks)**

e) A damped oscillator with mass $\mathbf{2kg}$, has the equation of motion $\mathbf{2\ddot{x} + 12\dot{x} + 50x = 0}$, where x is the displacement from equilibrium, measured in metres. What type of damping is this? Is the motion still oscillatory and periodic? If so, what is the oscillation period? **(4mrks)**

f) Consider a tractor being driven across a field that has undulations at regular intervals. The distance between the bumps is about $\mathbf{4.2m}$. Because of safety reasons, the tractor does not have a suspension system but the driver's seat is attached to a spring to absorb some of the shock as the tractor moves over rough ground. Assume the spring constant to be $\mathbf{2 \times 10^4 Nm^{-1}}$ and the mass of the seat to be $\mathbf{50kg}$ and mass of the driver is $\mathbf{70kg}$. The tractor is driven at $\mathbf{30kmh^{-1}}$ over the undulations. Will an accident occur? Explain. **(5 mrks)**

g) Your clock radio awakens you with a steady and irritating sound of frequency $\mathbf{600Hz}$. One morning, it malfunctions and cannot be turned off. In frustration, you drop the clock radio out of your fourth-storey dorm window, $15.0m$ from the ground. As you listen to the falling clock radio, what frequency do you hear just before you hear the radio striking the ground? Assume the speed of sound is $343ms^{-1}$ and $g = 9.81ms^{-2}$. **(4 mrks)**

h) The general solution to the equation of simple harmonic oscillation $\ddot{x} = -\omega^2 x$ is $x(t) = Asin\,(\omega t + \phi)$ for A and ϕ constants. Show that this can also be written as $x(t) = D_1 \cos(\omega t) + D_2 \sin\,(\omega t)$ with constants $D_1 = Asin\phi$ and $D_2 = Acos\phi$. Then show that this is equivalent to $x(t) = Z_1 e^{i\omega t} + Z_2 e^{-i\omega t}$ with complex constants $Z_1 = \frac{1}{2}(D_1 - iD_2)$ and $Z_2 = \frac{1}{2}(D_1 + iD_2)$. **(4mrks)**

i) Two sources, S_1 and S_2 emit harmonic waves in phase with the same amplitude, wavelength, and frequency. At a point P, which is a distance x_1 from source S_1 and a distance x_2 from source S_2, the equations of the two waves are $y_1(x_1, t) = Asin(kx_1 - \omega t)$ and $y_2(x_2, t) = Asin(kx_2 - \omega t)$. Use the standard trigonometric relation;

$sin\theta_1 + sin\theta_2 = 2cos\left(\frac{\theta_1-\theta_2}{2}\right) sin\left(\frac{\theta_1+\theta_2}{2}\right)$ to show that the total wave at P is;

$$y_{tot}(x, t) = 2Acos\,\left[k(\tfrac{x_1-x_2}{2})\right] sin\left[k\left(\tfrac{x_1+x_2}{2}\right) - \omega t\right]$$ **(2mrks)**

j) Under reflection and transmission of waves at an interface, you are given that $1 + R = T$ and $1 - R = \frac{v_1}{v_2}T$, where R is reflectance, T is transmittance and v_1, v_2 are the velocity of the wave in the original medium and the new medium respectively. Show that;

$$R = \frac{v_2 - v_1}{v_2 + v_1} \text{ and } T = \frac{2v_2}{v_2 + v_1}$$ **(2mrks)**

SECTION 2: ATTEMPT ANY TWO QUESTIONS

QUESTION 2: 20MRKS

a) Define the following terms. **(2mrks)**

A. Timbre

B. Forced oscillations

b) State any **three** sources of sound. **(3mrks)**

c) An object of mass $0.25kg$ hangs from a $6.3Nm^{-1}$ spring and is driven by a sinusoidal force with amplitude $1.7N$. What frequency will force the object to vibrate with amplitude of $0.4m$? **(3mrks)**

d) An instrument and its support system, which together act as under damped spring with spring constant, $6.3 \times 10^6 Nm^{-1}$ and mass, $318kg$ are meant to be installed on the Hubble Space Telescope. NASA requires that velocity resonance does not occur for forced oscillations at any frequency below $35Hz$. Does this instrument meet the requirement? If not, how might it be made to do so? **(4mrks)**

e) Two waves travelling in opposite directions produce a standing wave. The individual wave functions are;

$$y_1 = 4.0cm sin(3.0x - 2.0t)$$
$$y_2 = 4.0cm sin(3.0x + 2.0t)$$

where x and y are measured in centimetres.

Find the amplitude of the simple harmonic motion of the element of the medium located at $x = 2.3cm$. **(3mrks)**

f) Show that the total mechanical energy of a mass-spring system is $E_{tot} = \frac{1}{2}kA^2$, where the symbols used have their usual meaning. **(5mrks)**

QUESTION 3: 20MRKS

a) Define the following terms. **(2mrks)**

A. Restoring force

B. Doppler effect

b) Suppose an object is in simple harmonic motion about equilibrium, $x = 0$ and has an initial position x_o and an initial velocity v_o, at $t = 0$. If the angular frequency of oscillation is ω for this system, show that the amplitude and the phase constant are given by;

$$A = \frac{1}{\omega}(\omega^2 x_o{}^2 + v_o{}^2)^{1/2}$$

$$tan\phi = \frac{\omega x_o}{v_o}$$ **(5mrks)**

c) A particle of mass m is subjected to both a harmonic restoring force and a damping force that is linearly proportional to velocity. Hence, the total mechanical energy of the particle is $E_{tot} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m{\omega_o}^2x^2$ and its equation of motion is $m\ddot{x} + \gamma\dot{x} + m{\omega_o}^2x = 0$. Using the chain rule of differentiation, show that;

$$\frac{dE_{tot}}{dt} = \dot{x}(m\ddot{x} + m{\omega_o}^2x)$$

Hence, $\frac{dE_{tot}}{dt} = -\gamma\dot{x}^2$, so that the mechanical energy is not conserved. What is $-\gamma\dot{x}^2$ physically? **(5mrks)**

d) Show that the motion of a simple pendulum is **NOT** simple harmonic and show how it can be approximated to simple harmonic motion. **(5mrks)**

e) A pendulum has a period of $0.9927s$ in Tokyo, and $0.9924s$ in Cambridge. What is the ratio of g in Tokyo and Cambridge? **(3mrks)**

QUESTION 4: 20MRKS

a) An object of mass, $m = 25g$ on a frictionless flat surface is attached to the right hand end of a horizontal spring with spring constant, $k = 0.4Nm^{-1}$. At time $t = 0$, the object is located $10cm$ to the right of its equilibrium position and has a velocity $\dot{x} = 40cms^{-1}$ towards the right. Knowing that $x(t) = Asin(\omega t + \phi)$, find;

A. The angular frequency, ω of the oscillations. **(1mrk)**

B. The period, T. **(1mrk)**

C. The frequency , f. **(1mrk)**

D. The amplitude , A and the phase constant, ϕ. **(3mrks)**

b) Starting with the harmonic position function $D(x,t) = D_{max}cos(kx - \omega t)$, show that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency as;

$$I = \frac{1}{2}\rho v(\omega D_{max})^2$$

where the symbols used have their usual meaning. **(8mrks)**

c) The faintest sounds the human ear can detect at a frequency of $1000Hz$ correspond to an intensity of about $1.00 \times 10^{-12}Wm^{-2}$ (threshold of hearing). The loudest sounds the human ear can tolerate at this frequency correspond to an intensity of about $1.00Wm^{-2}$ (threshold of pain). Determine the pressure amplitude and displacement amplitude associated with the threshold of hearing. Explain your results. Take $v = 343ms^{-1}, \rho = 1.20kgm^{-3}$. **(4mrks)**

d) Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is $2.0 \times 10^{-7}Wm^{-2}$. Find the sound level heard by the worker when both machines are operating. Take $I_o = 1.00 \times 10^{-12}Wm^{-2}$. **(2mrks)**

QUESTION 5: 20MRKS

a) Define the term **beat** as used in the study of sound waves. **(2mrks)**

b) Two identical piano strings of length $0.750m$ are each tuned exactly to $440Hz$. The tension in one of the strings is then increased by 1.0%. If they are now struck, what is the beat frequency between the fundamentals of the two strings? **(3mrks)**

c) A section of drainage culvert $1.23m$ in length makes a howling noise when the wind blows.

   A. Determine the frequencies of the first three harmonics of the culvert if it is cylindrical in shape and open at both ends. **(3mrks)**

   B. What are the three lowest natural frequencies of the culvert if it is blocked at one end? **(3mrks)**

   (Take $v = 343ms^{-1}$ as the speed of sound in air.)

d) The equation of motion of a damped oscillator is $m\ddot{x} + \gamma\dot{x} + m\omega_o^2x = 0$ or $\ddot{x} + \frac{\gamma}{m}\dot{x} + \omega_o^2x = 0$, which has three classes of solution;

   I. $x(t) = e^{-(\gamma/2m)t}[C_1t + C_2]$

   II. $x(t) = A_oe^{-(\gamma/2m)t}sin(\omega t + \phi_o)$ with $\omega = \left[\omega_o^2 - \frac{\gamma^2}{4m^2}\right]^{1/2}$

   III. $x(t) = e^{-(\gamma/2m)t}[B_1e^{qt} + B_2e^{-qt}]$ with $q = \left[\frac{\gamma^2}{4m^2} - \omega_o^2\right]^{1/2}$

   A. What is the physical quantity represented by each term in the equation of motion? **(3mrks)**

   B. For each of the solutions I-III, name the type of damping that is described, give the criteria that determine whether the solution applies to a particular system, and sketch a typical $x(t)$ curve illustrating the main features of the motion. **(6mrks)**