MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2012/2013

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY (HOMA BAY CAMPUS)

SMA 201: LINEAR ALGEBRA II

Date: 27th July, 2013 *Time: 2.00 – 4.00 p.m.*

INSTRUCTIONS:

- Answer Question ONE and any other TWO questions.
- Start each question on a fresh page.
- Indicate question numbers clearly at the top of each page.

ISO 9001:2008 CERTIFIED

Question One

[Compulsory, 30mks]

(a) Define a vector space and show that $(\mathbb{Z}_5)^3$, the set of all vectors of length 3 is a vector space over the field $\mathbb{Z}_5$. (5mks)

(b) Define a linear transformation. Suppose that $T : U \to V$ is a linear transformation, show that $T(0) = 0$. (5mks)

(c) Define an eigenvector of a matrix A of size $n \times n$ and hence compute all the eigenvectors of the matrix

$$A = \begin{bmatrix} 2 & \frac{1}{2} \\ -1 & 3 \end{bmatrix}.$$

(5mks)

(d) Define a symmetric matrix and show that if a matrix A is symmetric then it is square. (5mks)

(e) Suppose that A is a square matrix with a row or a column where every entry is zero, prove that $\det(A) = 0$. (5mks)

(f) Suppose that A and B are invertible matrices, show that $(AB)^{-1} = B^{-1}A^{-1}$. (5mks)

Question Two

[20mks]

(a) Let

$$F = \{a + bt + ct^2 \mid a, b, c \in \mathbb{Z}_2\}.$$

Assume that the addition and multiplication of the elements of F are defined like those of polynomials in the variable t. If t^3 is replaced by $t + 1$, show that F is a field of size 8. (12mks)

(b) Suppose that A and B are $m \times n$ matrices such that $Ax = Bx$ for every $x \in \mathbb{C}^n$ show that $A = B$. (8mks).

Question Three

[20mks]

(a) Let $S_{22}(\mathbb{Z}_7)$ be the set of all symmetric 2×2 matrices over the field $\mathbb{Z}_7$. Prove that $S_{22}(\mathbb{Z}_7)$ is a vector space with the usual matrix addition and scalar multiplication and hence determine the number of matrices in $S_{22}(\mathbb{Z}_7)$ as well as $\dim(S_{22}(\mathbb{Z}_7))$. (10mks)

(b) (i) State Cayley-Hamilton theorem for a linear operator T.

(ii) If T is defined by $T([x, y]^t) = [x + 2y, 2x + y]^t$ write the matrix A representing T and use it to verify Cayley-Hamilton theorem.

(iii) Apply Cayley-Hamilton theorem to evaluate the polynomial

$$P(A) = A^4 - 7A^3 - 3A^2 + 4A + 4I.$$

(10mks)

Question Four

[20mks]

(a) Suppose that A is a square matrix, show that A has at least one eigenvalue. (10mks)

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation defined by $T([x, y, z]^t) = [4x + z, 2x + 3y + 2z, x + 4z]^t$. Find a matrix A representing T and determine an invertible matrix P and a diagonal matrix D such that $P^{-1}AP = D$. (10mks)

Question Five

[20mks]

(a) If A is a square matrix of size n and E is any elementary matrix of size n show that

$$\det(EA) = \det(E) \cdot \det(A).$$

(10mks)

(b) Given that $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear operator with matrix A of representation. If $T([x, y, z]^t) = [5x + y + 2z, -4x - 2z, -4x - y - z]^t$. Find all the eigenvalues and the corresponding eigenvectors of T. (10mks)

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