MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2015/2016

FIRST YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

CITY CAMPUS-SCHOOL BASED

SMA 103: LINEAR ALGEBRA I

Date: 12th February, 2016 Time: 2.00 - 4.00 pm

- Answer question ONE and any other TWO questions.

Question 1. **[Compulsory]** **[30 Marks]**

(a) Explain the meaning of the following as used in linear algebra

(i) a subspace of a vector space.

(ii) a linearly independent set.

(iii) a basis for a vector space **[6 Marks]**

(b) Solve the following system of linear equations using row reduction process.

$$-x_1 - x_2 + 2x_3 = 3$$

$$2x_1 + x_2 - 2x_3 = 5$$

$$x_1 + x_2 - x_3 = 1$$

[5 Marks]

(c) Given the vectors $\boldsymbol{u} = 2\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}$, $\boldsymbol{v} = 3\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}$. Find the acute angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ in degrees correct to one decimal place.

[5 Marks]

(d)Given the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, find the

(i) reduced row echelon equivalence of A.

(ii) basis for the column space of A.

(iii) rank of A. **[6 Marks]**

(e) If the points P=(1 , 1, 3) , Q = (-3 , -1, 4) and R = (-1, 3 , 7) are coplanar,

(i) find the symmetric equation of the line passing through P and Q.

(ii) determine the equation of the plane containing the three points.

(iii) compute to 2 decimal places, the area of the parallelogram whose consecutive vertices are P , Q and R. **[8 Marks]**

QUESTION 2.

(a) Let every vectors $\boldsymbol{u} = [x, y, z]$ in the subset V of $\mathbb{R}^3$ satisfy the following system of linear equations.

$$2x + y + 3z = 0$$
$$x + y + 2z = 0$$
$$y + z = 0$$

(i) Show that the vectors in V are of the form $\{\boldsymbol{v} = (-t, -t, t) : t \in \mathbb{R}\}$.

(ii) Show that V forms a vector subspace of $\mathbb{R}^3$ under the usual vector addition and scalar multiplication on the space $\mathbb{R}^3$.

(iii) Determine a basis and dimension of V.

(iv) Which of the following vectors belong to V?

$\boldsymbol{u_1} = [1, 2, 3]$, $\boldsymbol{u_2} = [0, 0, 0]$, $\boldsymbol{u_3} = [2, 2, -2]$ **[13 Marks]**

(b) Consider the system of linear equations;

$$x_1 + 2x_2 + x_3 = c_1$$
$$2x_1 + x_2 - x_3 = c_2$$
$$-4x_1 + x_2 + 5x_3 = c_3$$

(i) Given that c_1 , c_2 , c_3 are non zero scalars , find the condition they must satisfy if the system is consistent.

(ii)Determine the basis for the solution space. **[7 Marks]**

Question3.

(a) Let **u** and **v** be two vectors in $\mathbb{R}^3$. Show that $\|\boldsymbol{u}+\boldsymbol{v}\| \leq \|\boldsymbol{u}\| + \|\boldsymbol{v}\|$

[6 Marks]

(b) Consider the vectors $\boldsymbol{u} = -3i + j + 8k, \boldsymbol{v} = -2i - 3j + 5k$ and

$\boldsymbol{w} = 2i + 14j - 4k$ in $\mathbb{R}^3$

(i) Show that the vectors $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$ are coplanar.

(ii) Determine the equation of the plane containing $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$.**[8 Marks]**

(c) The planes $2x + 3y + 2z = 4$ and $x + 2y + 3z = 2$ intersect along some line L.

(i) Find the parametric equation of L.

(ii) Determine the directional vector of L and hence give the vector equation of L. **[6 marks]**

Question 4.

(a) (i) Let A be an $m \times n$ matrix and consider the mapping $T: \mathbb{R}^n \longrightarrow \mathbb{R}^m$ defined by $T(\boldsymbol{x}) = A\boldsymbol{x}$ for every n-vector $\boldsymbol{x}$. Show that T is a linear transformation.

(ii) If the range of T is $\{\boldsymbol{w}: \boldsymbol{w} = T(\boldsymbol{v})$ for some $\boldsymbol{v} \in \mathbb{R}^n.\}$. Show that range of T is a subspace of $\mathbb{R}^m$. **[12 Marks]**

(b) A linear map T from the vector space $\mathbb{R}^3$ into itself is defined by

$T([x, y, z]) = [x + y, y - z, x + z]$

(i) Find the matrix M_T which represents T with respect to the standard ordered basis for $\mathbb{R}^3$.

(ii) Find the Kernel of T

(iii) Determine the vectors which span the range of T. **[8 Marks]**

Question 5

(a) Let U and V be subspaces of $\mathbb{R}^3$ over a real field F.

(i) Show that $U \cap V$ is a vector subspace of $\mathbb{R}^3$.

(ii) Given that U = $\{[x, y, z] : x - y + z = 0\}$ and

V = $\{[x, y, z]: 2x + y - z = 0\}$, find the vector which spans $U \cap V$

[12 Marks]

(b) Let W be the subspace of $\mathbb{R}^4$ spanned by the set

$V = \{[1,2,1,1], [0,1,-1,1], [1,0,2,3], [1,-1,2,6]\}$.

(i) Show that V is a linearly dependent set.

(ii) Obtain a subset of V which forms a basis for W.

(iii) Verify that the vector $v = [1, 5, -1, 0]$ is a vector in W.

[8 MARKS]