MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2012/2013

SECOND YEAR SECOND SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL STUDIES WITH INFORMATION TECHNOLOGY (MAIN CAMPUS)

SAC 202: LIFE TESTING ANALYSIS

Date: 24th July, 2013 *Time: 8.30 – 10.30 a.m.*

INSTRUCTIONS:

- Attempt Question ONE (COMPULSORY) and any other TWO questions.

SAC 202: LIFE TESTING ANALYSIS

QUESTION ONE (30 MARKS)

(a). Write brief notes on the following concepts as used in survival analysis.

- (i). Complete expectation of life **(1mk)**
- (ii). Left censoring **(1mk)**
- (iii). Type I and Type II censoring **(2mks)**
- (iv). Integrated hazard function **(1mk)**

(b). (i). T_x denotes the future lifetime of a life currently aged x. Write down the probability density function of T_x. **(1Mk)**

(ii). Using your answer to (i), show that:

(a). $\frac{\delta}{\delta s} \log {}_sP_x = -\mu_{x+s}$

(b). ${}_tP_x = \exp\left\{-\int_0^t \mu_{x+s}\, ds\right\}$

(iii). In a certain population, the force of mortality is given by:

	μ_x
$60 < x \leq 70$	0.01
$70 < x \leq 80$	0.015
$X > 80$	0.025

Calculate the probability that a life aged exactly 65 will die between exact ages 80 and 83.**(3Mks)**

(iv) Give the Greenwood's formula and state its other name **(2mks)**

(c). (i). A scientist identifies 1,282 newborn wild beasts and observes them during their first year of life on the savannah. The scientist wishes to calculate the constant transition intensity (force of mortality) over this period covering all types of death, including natural causes and ending up as tasty snack for passing carnivores.

If the true transition intensify is 0.18, what is the probability that the scientist observes a hazard rate in excess of 0.2? **(5Mks)**

(ii). Give discrete hazard function. **(2Mks)**

(d) (i)In a mortality investigation covering a 5 – year period. Where the force of mortality can be assumed to be constant, there were 46 deaths and the population remained approximately constant at 7500. Estimate the force of mortality.

(ii) Prove that variance $(\mu) = \frac{\mu}{E^C}$ **(2MARKS)**

(e)(i) If Cox model only allows us to use time- independent covariates, does this mean that factors such as age, weight and severity of symptoms (which will vary with time) cannot be used? **(4mks)**

(ii) Give Gompertz Makeham family of curves from which we can model the force of mortality using one of the family curve (2mks)

QUESTION TWO (20MARKS)

(a) (i) Give and explain the Cox proportional hazard's model **(2mks)**

(ii) What are the main problems of using a parametric approach to analyze observed survival times? **(2mks)**

(iii) The covariate for the ith observed life are (56, 183, 40) representing (age last birthday at the start of the study, height in centimeters, daily dose of drug A in milligram (mg). Using regression parameters

$\beta = (0.0172, 0.0028, -0.0306)$, calculate $\lambda(t; Z_i$ in terms of λ_0 (t). **(2mks)**

(b) Losses arising from a certain group of policies are assumed to follow an $\exp(\lambda)$ distribution. You are given the following data;

- The exact amounts X_1, X_2, --, X_n paid by the insurer in respect of n losses.
- Data from a further m losses, in respect of which the insurer paid an amount "M" but you don't know by how much. Calculate the maximum likelihood estimate of λ. **(6mks)**

(ii) Define the actuarial symbols ${}_tP_x$ and ${}_tq_x$ and derive integral formulae for them. **(4mks)**

(iii) Give Gompertz's and Makeham's laws for the force of mortality **(4mks)**

QUESTION THREE

(a)(i) You are investigating the survival times of patients who have just undergone heart surgery at one of 3 city hospitals, -A, B or C. You have the following data for each patient.

$Z_1 = \begin{cases} 0 \text{ for females} \\ 1 \text{ for males} \end{cases}$

$Z_2 = \begin{cases} 1 \text{ if patient attended Hospital B} \\ 0 \text{ otherwise} \end{cases}$

$Z_3 = \begin{cases} 1 \text{ if patient attended hospital C} \\ 0 \text{ otherwise} \end{cases}$

You have decided to model the force of mortality at time t (measured in days since the operation was performed by an equation of the form $\lambda(t) = \lambda_0(t)e^{\beta z^T}$, and you have estimated the parameter values to be:

$\beta_1 = 0.031$, $\beta_{2=-0.025,}\beta_{3=0.011}$ compare the force of mortality for a female patient who attended Hospital A with that of

(i) A female patient who attended Hospital B

(ii) A male patient who attended Hospital C **(4mks)**

Define the expected value:

- Variance of the complete and curtate future lifetimes.
- Derive expressions for them
- Define the symbols e_x and $\dot{e}x$ and derive and approximately relation between them **(4mks)**

(b)(i) Describe the Kaplam Meier (product limit) estimate of the survival function in the presence of censoring.

(ii) Explain how the product limit arises as a maximum likelihood estimate.

(iii) Compute the estimate from a typical data

(iv) Estimate its variance **(8mks)**

(c) Define the curtate future lifetime from age X and state its probability. **(2mks)**

(d) Explain two concepts: Left censoring and left truncation **(2mks)**

QUESTION TOUR

(a)(i)Describe the estimation of the empirical survival function in the absence of censoring and what problems are introduced by censoring (4mks)

(ii) Give an example of a situation in which the hazard function may be expected to follow each of the following distributions:

i) Exponential

ii) Decreasing weibull

iii) Gompertz – Makeham

iv) Log logistic

(4mks)

(b)(i) If the jth covariates can take positive values only (e.g. age), what is the significance of

(1) The sigh of the jth regression parameters

(2) The magnitude of the jth regression parameters?

(6mks)

(ii) Describe the Nelson Aalen estimate of the cumulative hazard rate in the presence of censoring

- Explain how it arises as a maximum likelihood estimate
- Compute the estimate from typical data and
- Estimate its variance **(4mks)**

(iii) Explain the concept baseline hazard

(2mks)

QUESTION FIVE (20MKS)

(a)(i) Describe various ways in which lifetime data might be censored,

(8mks)

(ii) A group of lives was observed over a period of time as part of a mortality investigation. Each of the lives was under observation at all ages from age 55 until they died or were censored. The table below shows the sex, age at exit and reason for exit from the investigation.

Life	Sex	Age at exit	Reason for exit
1	M	56	Death
2	F	62	Censored
3	F	63	Death
4	M	66	Death
5	M	67	Censored
6	M	67	Censored

The following model has been suggested for the force of mortality

$\mu(x/Z \;\; = Z = \mu_{0\,(x)E^{\beta z}}$

Where:

- X denotes age
- $\mu_0(x)$ is the baseline hazard
- $z = 0$ for males and $z = 1$ for females

Write down the partial likelihood for these observations using the model above **(4mks)**

(b)(i) What is the difference between an estimate and estimator **(2mks)**

(ii) Explain the concept of censoring mechanism (2mks)

(iii)Losses arising from a certain group of policies are assumed to follow an experimental (λ distribution. You are given the following data;

(1) The exact amounts; X1,X2, - -, Xn paid by the insurer in respects for n losses
(2) Data from a further m losses, in respect of which the insurer paid an amount m.

The actual loss amounts exceeded M, but you don't know by how much. Obtain the MLE of λ **(4mks)**