MURANG'A UNIVERSITY COLLEGE

A constituent college of Jomo Kenyatta University of Agriculture and Technology

University Examination 2015/2016

THIRD YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2330: THEORY OF ESTIMATION

DATE: DECEMBER 2015

TIME: **2 HOURS**

Instructions: Attempt question **One** and **Two** other questions

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a) Define the following terms as used in theory of estimation

(i) A parameter and a statistic [2marks]

(ii) Sufficient statistic [2marks]

b) Let $x_1, x_2, \ldots, x_n$ be a random sample from Poisson distribution with parameter λ. Obtain by the method of moments the estimator for λ [5marks]

c) Let $x_1 x_2 \ldots x_n$ be a random sample from a Bernoulli population with parameter θ.

$$f(x) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0,1; 0 < \theta < 1 \\ 0 \ \ otherwise \end{cases}$$

Find the maximum likelihood estimator of θ. [4marks]

d) Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from a normal distribution with mean μ and variance σ^2. Show that the statistic defined by $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is an unbiased estimator of σ^2 where s^2 is the sampling mean whereas $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is not an unbiased estimator of σ^2.[7marks]

e) Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, 1)$ with mean μ and variance 1. Show that $T = \sum_{i=1}^{n} x_i$ is a sufficient statistic for μ using the Neyman Fisher factorisation criterion. [6marks]

f) Let $x_1, x_2, \ldots, x_n$ be a random sample from Poisson distribution with parameter $\lambda > 0$. Find the uniformly minimum variance unbiased estimator of λ. [4marks]

QUESTION TWO (20 MARKS)

a) Let X be a binomial random variable with parameter n (known) and p (unknown). Given a random sample of n observations of X, compute using the methods of moments the estimator for p. What would be the estimators when both parameters are unknown? [10marks]

b) Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, \sigma^2)$ where the mean μ and variance σ^2 are unknown parameters. Find the maximum likelihood estimators for:

(i) $\mu \ when \ \sigma^2$ is unknown [5marks]

(ii) $\sigma^2 \ when \ \mu$ is unknown [5marks]

QUESTION THREE (20 MARKS)

a) Let $x_1, x_2, \ldots, x_n$ be a random sample from a probability distribution given by;

$$f(x,\theta) = \begin{cases} \frac{2}{\theta^2}(\theta - x), & 0 < x < \theta, \quad 0 < \theta < \infty \\ 0, & otherwise \end{cases}$$

Obtain the estimate of θ by the method of moments [5marks]

b) Let $x_1, x_2, \ldots, x_n$ be a random sample from probability distribution given by;

$$f(x,\alpha) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0 \\ 0, & otherwise \end{cases}$$

Obtain the maximum likelihood estimator for parameter α [5marks]

c) Let $x_1, x_2, \ldots, x_n$ denote a random sample from a Beta distribution $f(x,\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma\alpha\Gamma\beta} x^{\alpha-1}(1-x)^{\beta-1}$ with parameter $\alpha > 0, \beta > 0$ unknown. Show that the family of Beta distribution with parameter $\alpha \; and \; \beta$ unknown belongs to a two parameter exponential family, hence determine jointly sufficient statistic for $\alpha \; and \; \beta$ [5marks]

d) Let $x_1, x_{2,}, x_3$ be a random sample from a size 3 from a distribution with mean θ and variance 100. Let $\hat{\theta} = \frac{1}{2}x_1 + \frac{1}{3}x_2 + \frac{1}{6}x_3$ be an estimator of θ. With respect to the squared-error loss function, find the risk of using $\hat{\theta}$ to estimate θ. [5marks]

QUESTION FOUR (20 MARKS)

a) State *without* proving the Cramer-Rao lower bound theorem including the regularity conditions assumed in proving it. [4marks]

b) Let $x_1, x_2, \ldots, x_n$ be a random sample from a Bernoulli distribution with parameter θ. If $\delta(\underline{x})$ is an unbiased estimator of θ. Find the lower bound of the variance of the unbiased estimator of θ, hence find an UMVUE of θ. [6marks]

c) Let $x_1, x_2, \ldots, x_n$ be a random sample from a *pdf* $f(x,\theta) = \theta^x(1-\theta), \quad x = 0,1,2 \ldots$. Show that the lowest possible variance for an unbiased estimator of $\frac{\theta}{1-\theta}$ is $\frac{\theta}{n(1-\theta)^2}$ [10marks]

QUESTION FIVE (20 MARKS)

a) Define the following terms as used in Bayesian statistics

(i) Prior distribution [3marks]

(ii) Posterior distribution [3marks]

b) Let $x_1, x_2, \ldots, x_n$ be a random sample from $N(\mu, 1)$. Let the prior distribution of μ be $N(0,1)$, that is $\lambda(\mu) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\mu^2}$. Obtain the Bayes estimator with respect to the squared error loss function [14marks]