



THE MOMBASA POLYTECHNIC UNIVERSITY COLLEGE

(A Constituent College of Jkuat)

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING (YR IV, SEM I)

SMA 2480: COMPLEX ANALYSIS

SPECIAL/SUPPLEMENTARY EXAMINATION

SERIES: FEBRUARY/MARCH 2012

TIME: 2 HOURS

Instructions to Candidates:

You should have the following for this examination

- Answer booklet
- Use SMP four figure mathematical tables and non-programmable electronic calculators.

This paper consists of **FIVE** questions

Answer question **ONE (COMPULSORY)** and any other **TWO** questions

This paper consist of **THREE** printed pages

QUESTION ONE (30 marks) compulsory.

a) Explain what is implied by the terms

- (i) Analytic function. (2 marks)
- (ii) Essential singularity (2 marks)

b) Show that $z = 2i$ is not a pole for $f(z) = \frac{z^2 + 4}{(z - 2i)(z + 3)}$. (3 marks)

(60 - 100j)Ω

c) An alternating voltage of 240V, 50Hz is connected across an impedance of .

Determine the magnitude of the impedance and its phase angl (2 marks)

$u = x^3 - 3xy^2$ v u $u + iv$

d) Given is harmonic find the harmonic conjugate of hence write in terms of z . (6 marks)

- e) Evaluate $\int_{1+4i}^{2+6i} (3x^2y + 2xy^2 - 3y + 2)dz$ along the straight line joining $1+4i$ to $2+6i$. (8 marks)
- f) Determine the linear fractional transformation T that maps $z_1=1, z_2=0, z_3=-1$ onto $w_1=i, w_2=\infty, w_3=1$, respectively. (7 marks)

QUESTION TWO (20 MARKS)

- a) Write down the equivalent polar form of $z = 6 - 8i$. (3 marks)
- b) Show that $f(z) = \cos\left(\frac{1}{z}\right) + 5z^{-2}$ is not analytic. (7 marks)

- c) Evaluate $\int_c \frac{3z+2}{(z^2+1)^3} dz$ where c is the circle $|z-2i|=2$ using Cauchy's integral formula. (10 marks)

QUESTION THREE (20 MARKS)

- a) If $w = f(z) = \frac{(2z+1)}{(3z-2)}$ where $z \neq \frac{2}{3}$ find the value of w corresponding to $z = 1+i$. (4 marks)

- b) Evaluate $\int_c \frac{2z^3 + 3z + 1}{(z+2)^2(z-3)} dz$ where c is any curve enclosing both $z = -2$ and $z = 3$. (8 marks)

- c) Evaluate $\int_{1+3i}^{2+6i} (4z^2 + 7z) dz$ along the straight line from $1+3i$ to $2+3i$ then from $2+3i$ to $2+6i$. (8marks)

QUESTION FOUR (20 MARKS)

- a) Find the poles of the function $f(z) = \frac{1}{z^2 + 2z + 5}$ then determine the residues at the poles. (6 marks)

$$f'(z) = \frac{1}{2\pi i} \int_c \frac{f(z)}{(z - z_0)^2} dz$$

b) Prove that (8 marks)

$$f(z) = z^3 + 2z + 5 - 3i$$

c) Show that the function is analytic everywhere on the z-plane. (6 marks)

QUESTION FIVE (20 MARKS)

$$\oint_c \frac{2z^2 + 1}{z^2 + 1} dz \quad c : |z + i| = 2$$

a) Find the integral of where (5 marks)

$$f(z) = \frac{z^3 - 4z^2 + 2i}{(z - 3i)^3} \quad \int_c f(z) dz$$

b) Find the residue of at its singularities hence evaluate where c is any simple closed curve containing all singularities of f(z). (8 marks)

$$\lim_{z \rightarrow (1+i)} \frac{z^2 - z + 1 - i}{z^2 - 2z + 2} = 1 - \frac{1}{2}i$$

c) Show that the (7 marks) THE END