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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF SCIENCE ACTUARIAL**

 **2016/2017 ACADEMIC YEAR**

**YEAR ONE SEMESTER TWO**

**MAIN REGULAR**

**APRIL 2017 EXAMINATION**

**COURSE CODE: SAS 102**

**COURSE TITLE: PROBABILITY AND DISTRIBUTION THEORY 1**

**EXAM VENUE: STREAM: (BSc. Actuarial)**

DATE: EXAM SESSION:

TIME: 2.00 HOURS

**Instructions:**

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the**

**examination room.**

**QUESTION ONE (COMPULSORY)-(30 MARKS)**

1. The joint density function of two continuous random variables X and Y is given by

$$f\left(x,y\right)=\left\{\begin{array}{c}k\left(2x-y\right), \&0\leq x\leq 2 ,0\leq y\leq 3\\0, \&otherwise\end{array}\right.$$

 Obtain

1. the value of $ k$.
2. the expected value of X (6marks)
3. Let $ f\left(x,y\right)=\left\{\begin{array}{c}6x^{2}y, \&0\leq x\leq 1 ,0\leq y\leq 1\\0, \&otherwise\end{array}\right.$

be the p.d.f. of two random variables *X* and *Y,* which must be of continuous type. Find

$P(0<x<^{3}/\_{4}, y>^{1}/\_{3})$ (6marks)

1. The joint probability function for two discrete random variables X and Y is tabulated as shown

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Y=0 | Y=1 | Y=2 | Y=3 |
| X=1 | 0.06 | 0.02 | 0.04 | 0.08 |
| X=2 | 0.15 | 0.05 | 0.10 | 0.20 |
| X=3 | 0.09 | 0.03 | 0.06 | 0.12 |

Determine

1. Marginal distributions of X and Y . (2marks)
2. P( X≤2, Y≥2) (2marks)
3. The failure of a circuit board interrupts work until a new board is delivered. The delivery time Y is uniformly distributed on the interval one to five days. The cost of a board failure and interruption C includes a fixed cost $C\_{0}$ and increases proportionally to the cube of the delivery time $Y^{3}$. This cost is modeled by $C=C\_{0}+C\_{1}Y^{3}$. Find
4. The probability that the delivery time does not exceed 4 days but must take at least one day.
5. In terms of $C\_{0}$ and$C\_{1}$, the expected cost associated with a single failed circuit board. (7marks)
6. Suppose X is a continuous random variable with pdf $f(x)=\left\{\begin{array}{c}5x^{4}, \&0<x<1\\0, \&otherwise\end{array}\right.$

Determine

1. The pdf of the continuous random variable Y where $Y=X^{3}$
2. $p(0.5<Y<1)$ (7marks)

**QUESTION TWO (20 MARKS)**

1. Given $f(x,y)=\left\{\begin{array}{c}2e^{-x-2y}, \&0<x<\infty ,0<y<\infty \\0, \&otherwise\end{array}\right.$ .

Determine

1. $P(X>1,Y<1)$
2. $P(X<Y=10)$ (9marks)
3. A random variable X has the Beta distribution with parameters $α$ and $β$ as shown below.

$f\left(x\right)=\left\{\begin{array}{c}\frac{Γ(α+β)}{ΓαΓβ}x^{α-1}\left(1-x\right)^{β-1}, \&0<x<1 ,α>0,β>0\\0, \&otherwise\end{array}\right.$

 Determine by derivation for this distribution, the standard deviation when $α=8, β=10$ . (11marks)

**QUESTION THREE (20 MARKS)**

1. The joint probability function of two discrete random variables X and Y is given by

$f\left(x,y\right)=\left\{\begin{array}{c}k\left(2x+y\right), \&0\leq x\leq 3 ,1\leq y\leq 3\\0, \&otherwise\end{array}\right.$

1. Obtain the value of $ k$.
2. Obtain $E(Y^{2})$
3. Deduce whether or not X and Y independent? (10marks)
4. Consider the Weibull distribution with parameters a and b

 $f\left(x\right)=\left\{\begin{array}{c}abx^{b-1}e^{-ax^{b}},x>0\\0, \&otherwise\end{array}\right.$

Obtain a general expression for the mean and the third raw moment for the distribution. (10marks)

**QUESTION FOUR (20 MARKS)**

1. The joint p.d.f of three continuous random variables X , Y and Z is defined as follows

$$f\left(x,y,z\right)=\left\{\begin{array}{c}k\left(xy+z\right), 0<\&x<3 , 0<y<4, 0<z<1\\0, \&otherwise\end{array}\right.$$

Calculate:

1. the value of k ,
2. the marginal distribution of X
3. $E({YZ}/{X=2)}$ (14marks)
4. Determine the value of c for which the function below is a joint probability density function.

$$f\left(x,y\right)=\left\{\begin{array}{c}c\left(x+y\right), 0<\&x<3 , x<y<2x+1\\0, \&otherwise\end{array}\right.$$

 (6marks)

**QUESTION FIVE (20 MARKS)**

1. A random variable Y has a probability density function given by $f\left(y\right)=\left\{\begin{array}{c}cy^{3}e^{-{y}/{2}}, \&y>0,\\0, \&otherwise\end{array}\right.$.

 Find C hence show that Y has a chi-square distribution. State the degrees of freedom. (10 marks)

1. Let X and Y be two independent standard normal random variables. Suppose $U=X+Y $and $V=2X-Y$ are two new random variables in terms of X and Y. Determine the joint pdf of U and V. (10 marks)