TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR DEGREE OF:

BACHELOR OF SCIENCE IN CIVIL ENGINEERING
BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING
BACHELOR OF SCIENCE IN ELECTRICAL & ELECTRONIC ENGINEERING

SMA 2270/SMA 2277: CALCULUS III

END OF SEMESTER EXAMINATION
SERIES: DECEMBER 2014
TIME ALLOWED: 2 HOURS

<u>**Instructions to Candidates:**</u>
You should have the following for this examination
- *Mathematical tables*
- *Scientific Calculator*

This paper consist of **FOUR** questions
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

Question One (Compulsory)

a) Evaluate $\lim_{x \to 0^+}\left(\dfrac{1}{\sin^2 x} - \dfrac{\cot x}{x}\right)$ (4 marks)

b) Show that the function $f(x) = x - x^3$ on the interval $-1 \le x \le 0$ and $0 \le x \le 1$ satisfies Rolle's theorem. Find the appropriate value of C **(5 marks)**

c) Compute the fifth order Taylor's polynomial P^5 at a = 1 for the function $f(x) = \dfrac{1}{x}$ **(5 marks)**

$$\{b_n\} = \frac{2n}{1+n}$$

d) Determine whether the following sequence is monotonic or not and if it is bounded

(3 marks)

$$x_n = \frac{2n^3 - 3n}{5n^3 + 4n^2 - 2} \qquad n \to \infty$$

e) Evaluate the limit of the sequence as **(3 marks)**

$$\int \cos^8 x \, dx$$

f) Evaluate by reduction formula **(5 marks)**

$$\int_1^3 \int_{x^{1/3}}^x \frac{y^2}{x} \, dy \, dx$$

g) Evaluate iterated double integral **(5 marks)**

Question Two

$$\lim_{x \to 0^+} x^x$$

a) Evaluate **(5 marks)**

$$\sin 45^\circ = \frac{1}{\sqrt{2}} \qquad \cos 45^\circ = \frac{1}{\sqrt{2}}$$

b) Given and approximate sin 44º by use of a Taylor's series expansion up to x^3 **(5 marks)**

$$f(x) = \frac{k}{(1+x^2)}$$

$$(-\infty, \infty)$$

c) The probability density function has the area under the curve on the interval equal to 1, determine the value of K. **(6 marks)**

$$\frac{dz}{dt} \qquad Z = \ln(x^2 + y^2) \qquad x = e^{-t} \qquad y = e^t$$

d) Find if given and **(4 marks)**

Question Three

$$\sum_{n=1}^{\infty} \frac{1}{2^n}$$

a) Find the nth partial sum of the series hence the sum to infinity **(4 marks)**

$$r = 3\cos\theta$$

b) Determine by integration the area of the region lying inside the circle on the positive x-axis

$$r = 1 + \cos\theta$$

and outside the cardiod **(6 marks)**

$$\{a_n\} \qquad \frac{2}{1}, \frac{4}{3}, \frac{8}{5}, \frac{16}{7}, \frac{32}{9}, \ldots$$

c) Find a sequence whose first five terms are hence determine whether it converges or diverges **(5 marks)**

$$f(x) = \sin x$$

d) Represent $f(x) = \sin x$ as the sum of its Taylor series centred at $\dfrac{\pi}{3}$ **(5 marks)**

Question Four

a) Explain what is meant by continuity of a function, hence determine whether the function $f(x) = \dfrac{1}{x}$ is continuous within the interval (0, 1) **(6 marks)**

b) Two police road-blocks are 10km apart on a highway. As a car passes the first road-block its speed is clocked at 60kmh^{-1}. Five minutes later when the car passes the second road block, its speed is clocked at 45kmh^{-1}. Prove that the car must had exceed the speed limit of 100kmh^{-1} at some point during the five minutes. **(6 marks)**

c) Using double integration, determine the volume of the solid generated by revolving the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ about x – axis **(8 marks)**

Question Five

a) Use ratio test to determine whether the series $\sum \dfrac{n!\,n!}{(2n)!}$ converges or diverges **(5 marks)**

b) Determine the surface area generated by revolving the circle $x^2 + y^2 = 6^2$ about the x-axis **(5 marks)**

c) Determine the Cartesian equation for the curve $r\cos\left(\theta - \dfrac{\pi}{3}\right) = 3$ **(4 marks)**

d) Find the volume in the first octant between the planes z = 0 and $z = x + y + z$ and inside the cylinder $x^2 + y^2 = 16$ **(6 marks)**