TECHNICAL UNIVERSITY OF MOMBASA

Faculty of Applied & Health Sciences

DEPARTMENT OF MATHEMATICS & PHYSICS

UNIVERSITY EXAMINATION FOR THE BACHELOR OF SCIENCE IN MECHANICAL ENGINEERING/CIVIL ENGINEERING

SMA 2370: CALCULUS IV

SPECIAL/SUPPLEMENTARY EXAMINATION
SERIES: OCTOBER 2013
TIME: 2 HOURS

<u>Instructions to Candidates:</u>
You should have the following for this examination
- *Answer Booklet*

This paper consist of **FIVE** questions in **TWO** sections **A** & **B**
Answer question **ONE (COMPULSORY)** and any other **TWO** questions
Maximum marks for each part of a question are as shown
This paper consists of **THREE** printed pages

SECTION A (COMPULSORY)

Question One

a) Show that the function,

$$f(x,y) = \begin{cases} \dfrac{2xy}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

is continuous at every point of the junction except the origin **(6 marks)**

b) Given the function $h(x,y) = x^3 + y^3$ where x = r – s and y = r + s, find:

(i) $\frac{\partial u}{\partial r}$

(ii) $\frac{\partial u}{\partial s}$ **(7 marks)**

c) Find the directional derivative of $g(x, y) = x^2 + e^{xy} + 2xz$ at (1, 0, 1) in the direction of the line segment $\vec{PQ}$ where P (0, 2, 4)

d) Find the equation of the tangent plane and the normal line of the surface $f(x, y, z) = x^2y^2 + z - 9 = 0$, at $x_o(1,2,4)$ **(6 marks)**

e) Show that the integral:

$\int_0^1 \frac{\sec x}{x} dx$ is divergent **(5 marks)**

SECTION B (Answer any TWO questions from this section)

Question Two

a) Find the stationary values of $x^2 + y^2 + z^2$ given that $ax + by + cz = \rho$ using Langrange multipliers. **(6 marks)**

b) A rectangular box open at the top is to have a capacity of 108m³. Find the dimensions of the box requiring the least material for its construction **(7 marks)**

c) Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane z = 4 **(7 marks)**

Question Three

a) Find the volume of the prism whose base is the triangle in the xy plane bounded by the x-axis and the the line the plane z = 3 – x –y **(8 marks)**

b) Verify Green's theorem in the plane for $\oint (2xy - x^2)dx + (x + y^2)dy$ where c is the closed curve of the region bounded by $y = x^2,\ y^2 = x$ **(12 marks)**

c) Find the area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ **(6 marks)**

Question Four

a) State Stoke's theorem both in words and in equation form **(2 marks)**

b) Verify Stoke's theorem for the vector field $\vec{F} = x\underset{\sim}{i} + y\underset{\sim}{j} + 2xy\underset{\sim}{k}$ using the hemisphere $x^2 + y^2 + z^4,\ z \neq 0$

(12 marks)

c) Find the area of the region bounded by the line $y = 3x$ and the curve $y = x^2$ (6 marks)

Question Five

a) Given that g is a function of two variables defined by:

$$g(x, y) = \frac{x^3 + y^3}{x - y}, x \neq y, g(x, y) = 0$$

when x = y show that g is discontinuous at the origin but the first order partial derivative exists at that point **(6 marks)**

b) Find an equation for the tangent to the ellipse:

$$\frac{x^2}{4} + y^2 = 2$$

at the point (-2, 1) **(5 marks)**

c) Expand the Fourier series the function f(x) sketched below: **(9 marks)**

1