



EMBU UNIVERSITY COLLEGE
(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE,
BACHELOR OF EDUCATION (SCIENCE/ARTS), BACHELOR OF SCIENCE
(INDUSTRIAL CHEMISTRY), BACHELOR OF SCIENCE (ANALYTICAL
CHEMISTRY) AND BACHELOR OF SCIENCE (STATISTICS)

SMA 104: CALCULUS II

DATE: APRIL 12, 2016

TIME: 02:00-04:00

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE

- a) Define a monotone function and find the intervals of monotonicity of the function

$$f(x) = x^3 - 6x^2 + 4 \quad (5 \text{ Marks})$$

- b) Investigate the given function for points of extremum by use of first derivative

$$f(x) = -x^2 + 8x + 2 \quad (4 \text{ Marks})$$

- c) Find the Taylor polynomial of degree 5 for the function

$$f(x) = \sin x, \text{ about } x = 0 \quad (4 \text{ Marks})$$

d) Evaluate the following indefinite integral

$$\int e^{x^3 + \cos x} (3x^2 - \sin x) dx \quad (4 \text{ Marks})$$

e) Determine the following integral

$$\int \frac{\cos^2 x}{1 - \sin x} dx \quad (4 \text{ Marks})$$

f) Find the length of the arc from $\theta = 0$ to $\theta = \frac{\pi}{4}$ of the curve given by

$$x = 3 \cos \theta \quad y = 3 \sin \theta \quad (5 \text{ Marks})$$

g) C is a curve given by $y = \sin x$ for values of x between 0 and π , find the area enclosed by the region C and the x-axis.

(4 Marks)

QUESTION TWO

a) When is a function $y = f(x)$ said to be

i.) Increasing and decreasing in an interval (a, b) ? (2 Marks)

ii.) Convex upwards and convex downwards on the interval $[a, b]$? (2 Marks)

b) Consider the function $y = x^4 - 3x^3$. On what intervals is it increasing and decreasing?

On what intervals is it convex downward and upwards?

Find and classify the relative extrema and points of inflection. Sketch the graph.

(10 Marks)

c) For the function $f(x) = x^2 + 2$ and $g(x) = x^3 - 1$, test whether the Cauchy's theorem holds on the interval $(1, 2)$ and find the appropriate value of ξ (6 Marks)

QUESTION THREE

- a) Using the second derivative, find out the nature of extrema of the following function:

$$f(x) = x^3 - 9x^2 + 24x - 12 \quad (5 \text{ Marks})$$

- b) Evaluate the following indefinite integrals

i.) $\int x^2 e^{2x} dx$ (5 Marks)

ii.) $\int \cos 12x \sin 3x dx$ (5 Marks)

c) Prove that $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$ (5 Marks)

QUESTION FOUR

- a) Resolve $\frac{3x^2 - 2x - 7}{x^2 - x - 2}$ into partial fractions .

Hence evaluate $\int \frac{3x^2 - 2x - 7}{x^2 - x - 2} dx$ (10 Marks)

- b) Sketch the region enclosed by the curve $y = x^3 - 3x^2 + 2x$, the x-axis and the ordinates $x = 0$ to $x = 2$. Hence, find the area of the enclosed region.

(10 Marks)

QUESTION FIVE

- a) Applying integration methods, find the volume of a cone of height h and radius r , using the relation

$$y = \frac{r}{h}x, \text{ within the interval } 0 \leq x \leq h$$

(10 Marks)

- b) Show that the function $f(x) = x^3 + 1$ on the interval $1 \leq x \leq 2$ satisfies the Lagrange's theorem. Find the appropriate value of $x = c$

(5 Marks)

- c) State Rolle's Mean Value Theorem of the differential calculus. Hence determine the value $x = c$ whose existence is guaranteed by Rolle's theorem for the function

$$y = f(x) = x^2 - 2x - 8 \text{ on } -2 \leq x \leq 4$$

(5 Marks)

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