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EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2013/2014

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 104: CALCULUS II

DATE: DECEMBER 9, 2013 **TIME: 11.00 – 1.00PM**

INSTRUCTIONS:

ANSWER QUESTION **ONE** AND ANY OTHER **TWO** QUESTIONS

QUESTION ONE (30 MARKS)

(a) Evaluate the following integrals:

(i) $\int_0^{\frac{\pi}{2}} \sin 7x \cos 6x dx$ (3marks)

(ii) $\int x^2 \sqrt{x^3 - 7} dx$ (3marks)

(iii) $\int x \ln x dx$ (3marks)

(iv) $\int \frac{2}{x^2 - 1} dx$ (4 marks)

(b) Find the area enclosed by the curves $y = x^3 - x$, the x-axis between $x = 0$ and $x = 1$ (3marks)

(c) (i) State the First Fundamental Theorem of calculus. (2marks)
Hence evaluate:

(ii) $\int_{0}^{\frac{\pi}{2}} \cos^2 x dx$ (3 marks)

(iii) $\int_{-1}^{1} (x^3 - 2x^2 - x + 2) dx$ (3marks)

(d) A particle starts from rest at $t = 0$ and moves so that at any time t seconds its acceleration is given by $a = t(8 - 3t) units$. Find at what time it again comes to rest and the distance it has moved from start. (6marks)

QUESTION TWO (20 MARKS)

(a) The velocity of a train after leaving a station is given as follows:

Time in min	0	2	4	6	8	10	12	14	16
Speed in meters/min	0	50	110	160	230	290	360	410	470

Use Simpson's rule to find the distance traveled in the first 16 minutes. (7marks)

(b) Find the length of the arc of the curve $y = \sqrt{x^3}$ from $x = 1$ to $x = 4$. (5marks)

(c) Express $\dfrac{3x+1}{(x-1)(x^2+1)}$ into partial fractions and hence evaluate $\int \dfrac{3x+1}{(x-1)(x^2+1)} dx$

(8marks)

QUESTION THREE (20 MARKS)

(a) (i) Find the 5^{th} degree Taylor series for $f(x) = \ln x$ at $x = 1$ (6marks)

(ii) Use the answer in part (a) (i) above to approximate $\ln 0.6$ (2marks)

(b) Find the area enclosed by the curves $y = x^2 - 4x + 2$ and $y = 2 - x^2$ (5marks)

(c) Evaluate the following:

(i) $\int x^2 e^{2x} dx$ (4marks)

(ii) $\int \dfrac{1}{1+\sin^2 x} dx$ (4marks)

QUESTION FOUR (20 MARKS)

(a) Evaluate (a) $\int \frac{dx}{\sqrt{x^2+4}}$ (4marks)

(b) $\int_0^1 (x^5 - 4x^3 + 3x - 2)dx$ (3 marks)

(c) (i) State the Mean Value Theorem. (2marks)

(ii) Verify the Mean Value Theorem for the function $y = x^2 - 2x$ on the interval (0, 2). (4marks)

(d) Use the Trapezoidal rule to approximate $\int_0^{\pi/2} \sqrt{\sin x}dx$ with n=6 where x is measured in radians and find the error in the approximation. (6marks)

QUESTION FIVE (20 MARKS)

(a) The area bounded by $y = x^3$, $x = 2$, x-axis and $x = 0$ is rotated about the x-axis. Find the volume of the solid of revolution. (5marks)

(b) Find the surface area of the solid generated by rotating the curve defined by $x = 3\cos\theta$, $y = 3\sin\theta$ on the interval $0 \le \theta \le \frac{\pi}{4}$ about the x-axis. (5marks)

(c) Evaluate the following integrals:

(i) $\int \frac{\cos 2x}{\sin^3 2x}dx$ (4marks)

(ii) $\int \frac{x^3 - 6x^2 + 5x - 3}{x^2 - 1}dx$ (6marks)

DEFINITIONS AND FORMULAE

Indefinite integrals of common functions

[In the following we take $a > 0$ and omit the additive constant.]

$f(x)$	$\int f(x)dx$
$x^n\ (n \neq -1)$	$x^{n+1}/(n+1)$
$1/x$	$\ln x$ if $x > 0$, $\ln(-x)$ if $x < 0$ (i.e. $\ln\lvert x\rvert$, $x \neq 0$)
$\dfrac{1}{x^2+a^2}$	$\dfrac{1}{a}\tan^{-1}\dfrac{x}{a}$
$\dfrac{1}{x^2-a^2}$	$\dfrac{1}{2a}\ln\left\lvert\dfrac{x-a}{x+a}\right\rvert$
$\dfrac{1}{\sqrt{(x^2+a^2)}}$	$\sinh^{-1}\dfrac{x}{a}$
$\dfrac{1}{\sqrt{(x^2-a^2)}}$	$\cosh^{-1}\dfrac{x}{a}$ if $x > a$, $-\cosh^{-1}\left(\dfrac{-x}{a}\right)$ if $x < -a$
$\dfrac{1}{\sqrt{(a^2-x^2)}}$	$\sin^{-1}\dfrac{x}{a}$
$\sin x$	$-\cos x$
$\cos x$	$\sin x$
$\tan x$	$\ln\lvert\sec x\rvert$
$\cot x$	$\ln\lvert\sin x\rvert$
$\sec x$	$\ln\lvert\sec x+\tan x\rvert = \ln\lvert\tan(\frac{1}{2}x+\frac{1}{4}\pi)\rvert$
$\csc x$	$\ln\lvert\tan\frac{1}{2}x\rvert$
$e^{ax}\sin bx$	$\dfrac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
$e^{ax}\cos bx$	$\dfrac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$
$\sin^2 x$	$\frac{1}{2}(x-\frac{1}{2}\sin 2x)$
$\cos^2 x$	$\frac{1}{2}(x+\frac{1}{2}\sin 2x)$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$

For logarithmic forms of inverse hyperbolic functions see p. 3.

Integration by parts

$$\int u\frac{dv}{dx}\,dx = uv - \int\frac{du}{dx}\,v\,dx.$$

Reduction formulae for trigonometric integrals

$$\int_0^{\frac{1}{2}\pi}\sin^m x\,dx = \frac{m-1}{m}\int_0^{\frac{1}{2}\pi}\sin^{m-2} x\,dx;\quad \int_0^{\frac{1}{2}\pi}\cos^m x\,dx = \frac{m-1}{m}\int_0^{\frac{1}{2}\pi}\cos^{m-2} x\,dx;$$

$$\int_0^{\frac{1}{2}\pi}\sin^m x\cos^n x\,dx = \frac{m-1}{m+n}\int_0^{\frac{1}{2}\pi}\sin^{m-2} x\cos^n x\,dx = \frac{n-1}{m+n}\int_0^{\frac{1}{2}\pi}\sin^m x\cos^{n-2} x\,dx.$$

[These results hold provided that the exponents in the reduced form are greater than -1. There are analogous reduction formulae with other intervals of integration $(\frac{1}{2}k_1\pi, \frac{1}{2}k_2\pi)$ with k_1, k_2 integral.]

DEFINITIONS AND FORMULAE

Area and volume formulae

Volume of a cone or pyramid $= \frac{1}{3}Ah$, where A = base area, h = height of vertex.
Area of curved surface of a cone $= \pi r l$, where l = slant height, r = base radius.
Volume of a sphere $= \frac{4}{3}\pi r^3$.
Surface area of a sphere $= 4\pi r^2$.
Area of a spherical zone
(between planes distance h apart) $= 2\pi r h$.

Trigonometry

$$\sec\theta = \frac{1}{\cos\theta}; \quad \csc\theta = \frac{1}{\sin\theta}; \quad \tan\theta = \frac{\sin\theta}{\cos\theta}; \quad \cot\theta = \frac{1}{\tan\theta}.$$

$$\cos^2\theta + \sin^2\theta = 1; \quad 1 + \tan^2\theta = \sec^2\theta; \quad \cot^2\theta + 1 = \csc^2\theta.$$

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi; \quad \cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi;$$

$$\tan(\theta \pm \phi) = \frac{\tan\theta \pm \tan\phi}{1 \mp \tan\theta\tan\phi} \quad [\theta \pm \phi \neq (k + \tfrac{1}{2})\pi].$$

$$\sin 2\theta = 2\sin\theta\cos\theta; \quad \cos 2\theta = \cos^2\theta - \sin^2\theta; \quad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta} \quad [\theta \neq (\tfrac{1}{2}k + \tfrac{1}{4})\pi].$$

$$2\cos^2\theta = 1 + \cos 2\theta; \quad 2\sin^2\theta = 1 - \cos 2\theta.$$

If $t = \tan\frac{1}{2}\theta$, then $\sin\theta = \frac{2t}{1+t^2}$; $\cos\theta = \frac{1-t^2}{1+t^2}$; $\tan\theta = \frac{2t}{1-t^2}$; $\frac{d\theta}{dt} = \frac{2}{1+t^2}$.

$$2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi);$$

$$2\cos\theta\cos\phi = \cos(\theta+\phi) + \cos(\theta-\phi);$$

$$2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi).$$

$$\sin\alpha + \sin\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta); \quad \sin\alpha - \sin\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\alpha-\beta);$$

$$\cos\alpha + \cos\beta = 2\cos\tfrac{1}{2}(\alpha+\beta)\cos\tfrac{1}{2}(\alpha-\beta); \quad \cos\alpha - \cos\beta = 2\sin\tfrac{1}{2}(\alpha+\beta)\sin\tfrac{1}{2}(\beta-\alpha).$$

In the triangle ABC:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R;$$

$$a^2 = b^2 + c^2 - 2bc\cos A, \text{ etc.};$$

$$\tan\tfrac{1}{2}A = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}, \text{ etc.}; \quad \text{area} = \sqrt{\{s(s-a)(s-b)(s-c)\}};$$

where $s = \frac{1}{2}(a+b+c)$.

Ranges of the inverse functions:

$$-\tfrac{1}{2}\pi \leqslant \sin^{-1}x \leqslant \tfrac{1}{2}\pi; \quad 0 \leqslant \cos^{-1}x \leqslant \pi; \quad -\tfrac{1}{2}\pi < \tan^{-1}x < \tfrac{1}{2}\pi.$$