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EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2013/2014

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 103: CALCULUS I

DATE: DECEMBER 9, 2013 **TIME: 8.30 – 10.30**

INSTRUCTIONS:

ANSWER QUESTION **ONE** AND ANY OTHER **TWO** QUESTIONS

QUESTION ONE (30 MARKS)

(a) Determine the domain of the function $f(x)=\sqrt{x^2-x-12}$ (2 marks)

(b) Evaluate $\lim_{x\to\infty}\sqrt[3]{\frac{x^2+5}{125x^2-7}}$ (3marks)

(c) If $f(x)=\cos x$, use the first principles to show that $f'(x)=-\sin x$ (4 marks)

(d) If $e^x y=\sin x$, show that $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=0$ (4marks)

(e) Using $y=\sqrt{x}$, estimate the value of $\sqrt{99}$ without the use of a calculator. (3marks)

(f) Sketch the graphs of the equations $x+y=0$ and $x=\sin y$ and show that they are orthogonal. (4 marks)

(g) Consider the function $f(x)=\begin{cases} x & 0\le x<1 \\ \frac{1}{2}x & 1\le x<2 \end{cases}$

Determine whether $f(x)$ is continuous or not at $x=1$ (3marks)

(h) Determine the slope of the tangent to the graph $x^2+4y^2=7$ at the point $(\sqrt{2},\frac{-1}{\sqrt{2}})$ (3marks)

(i) A glass which has the shape of a cone of height 20cm and base radius 4cm is being filled from a tap at the rate of 25cm/sec.How fast is the level of water rising at the instant when the height of water in the glass is 10cm? (4 marks)

QUESTION TWO (20MARKS)

(a) Find the derivatives of the following functions:

(i) $y=\tan^{-1}2x$ (2marks)

(ii) $x^3+xy^2-y^3=9$ (3marks)

(iii) $y=\log_a x^2$ (3marks)

(iv) $y=\ln e^{3x}$ (3marks)

(b) Using the first principles, show that $\frac{d}{dx}(\ln x)=\frac{1}{x}$ (5marks)

(c) Find $\frac{dy}{dx}$ for the given parametric equations:

$x=\sin t, y=\cos 2t.$

Hence show that $\frac{d^2y}{dx^2}+4=0$ (4marks)

QUESTION THREE (20MARKS)

(a) Find the equation of :

(i) the normal and

(ii) the tangent to the curve $y=\frac{4}{x^2}$ at the point where $x=1$. (4marks)

(b) Find the coordinates of the points on the curve $y=x^3-6x^2+12x+2$ at which the tangent is parallel to the line $y=3x$ (4marks)

(c) Sketch the intersecting graphs of the equations below and show that they are orthogonal.

$2x^2+y^2=6$ and $y^2=4x$ (6marks)

(d) Find the area enclosed by the curve y=x(x-2) and the line y=-x+2. (6marks)

QUESTION FOUR (20 MARKS)

(a) If $y = \sin^{-1}(\tanh x)$, show that $\frac{dy}{dx} = \sec hx$ (5marks)

(b) Distinguish between an odd and an even function. Hence determine whether the function $f(x) = x^7 + x$ is odd or even. (3marks)

(c) Find the domain and range of the function $y = \sqrt{1 - x^2}$ (3marks)

(d) Evaluate $\lim_{x \to 0} \frac{\sin 3x}{\sin 4x}$ (4marks)

(e) Find the equation of the tangent to the curve $y = x^2 - 6x + 5$ at each point where it crosses the x-axis. Find also the coordinates of the points where these tangents meet. (5marks)

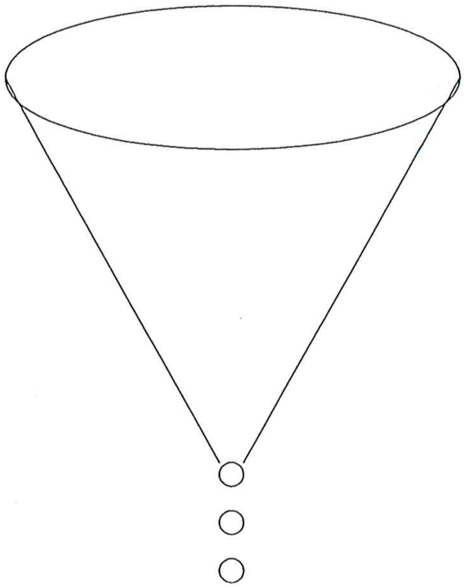
QUESTION FIVE (20 MARKS)

(a) (i) Find the interval for which $f(x) = x^3 - \frac{3}{2}x^2$ is increasing or decreasing. (3marks)

(ii) Using the second derivative test, find the relative extrema for the function $f(x) = -3x^5 + 5x^3$ (4marks)

(b) A spherical balloon is being blown up such that its volume increases at the rate of $1.5cm^3 / \sec$ .Find the rate at which its radius increases when the volume of the balloon is $56cm^3$. (5marks)

(c) A funnel holding a liquid has the shape of an inverted cone with a semi-vertical angle of 30° .The liquid is running out of a small hole at the vertex.



(i) If the hole is small enough to be ignored when finding the volume, show that the volume Vcm^3 of the liquid left in the funnel when the depth of the liquid is h cm is given by $\frac{1}{9}\pi h^3$ (4marks)

(ii) If it is further assumed that the liquid is running out at a constant rate of $3cm^3/\sec$, find the rate of change of h when $V = 81\pi cm^3$. (4marks)

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