MURANGA UNIVERSITY COLLEGE

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2101 CALCULUS I

DATE: DECEMBER 2013 **TIME 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a) Prove the following identities

i. Tan (A – B) = $\frac{\tan B - \tan A}{1 + \tan A \tan B}$ (3 marks)

ii. Tan 3A = $\frac{3\tan A - tan^3 A}{1 - 3tan^2 A}$ (3 marks)

b) Find the area under the curve y = x^3 + 2x over interval [0, 1] (2 marks)

c) Evaluate the limits below

a) $\lim_{x \to 3}(\frac{x+1}{x^2-2})$ (2 marks)

b) $\lim_{x \to -1}(\frac{x(x+1)^2}{x} + 4x^3 + 3)$ (2 marks)

d) Examine the continuity of the functions below

a) f(x) = sin x as x → 0 (2 marks)

b) f(x) = $\frac{x^2-1}{x+1}$ as x → 1 (2 marks)

e) A manufacturer of canned fruits uses tins with approximate rectangular bases with sides in the ratio 3 : 2 holding 30 cms^3 each. What height should the tins be to use as little metal as possible? (4 marks)

f) Find $\frac{dy}{dx}$ given y = $\frac{x^5}{\sqrt{x+1}}$ (4 marks)

g) Evaluate the integrals $\int_0^{10} x^3 dx$ using the Simpsons rule, five divisions. (3 marks)

h) Evaluate the following integral $\int sin^2 x \, dx$ (3 marks)

QUESTION TWO (20 MARKS): OPTIONAL

a) Use first principles to prove that if $f(x) = \cos x$, then $f^1(x) = -\sin x$ (6 marks)

b) The acceleration of a particle t seconds after the start is $(8 - 6t^2)$ m/s^2. Find the velocity and the displacement of the particle in terms of t given that the particle started with a velocity of 2m/s. How far is the particle from the starting point after 2 seconds and what is its velocity? (8 marks)

c) Evaluate the integrals below

i. $\int \frac{dx}{\sqrt{3-x^2}}$ (3 marks)

ii. $\int e^{x-2}\, dx$ (3 marks)

QUESTION THREE (20 MARKS): OPTIONAL

a) Show that the gradient of the curve $y = 3\sin\theta - \sin^3\theta$, $x = \cos^3\theta$ is $-\cot\theta$ (5 marks)

b) Find the equation of the tangent and normal to the curve

$x^2 + 2xy + 3y^2 = 4$ at the point where x = 3 (5 marks)

c) Find the area enclosed by the curves $y = x^2 - 5x$ and $y = 3x^2 - 6x$ (10 marks)

QUESTION FOUR (20 MARKS): OPTIONAL

a) Find $\frac{dp}{dk}$ given that

i. $p = e^{\tan (k^2+1)}$ (5 marks)

ii. $p = \ln \frac{k}{(k^2+1)^{\frac{1}{2}}}$ (7 marks)

b) Given the following inverse trigonometric evaluate their derivatives.

i. $y = \sin^{-1}3x$ (4 marks)

ii. $y = \tan^{-1}3x$ (4 marks)