MURANGA UNIVERSITY COLLEGE

LOGO !!!

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF

BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

SMA 2100 : DISCRETE MATHEMATICS

DATE: DECEMBER 2013 **TIME 2 HOURS**

INSTRUCTIONS: ANSWER QUESTION ONE (COMPULSORY) ANY OTHER TWO QUESTIONS

QUESTION ONE (30 MARKS)

a)

i. Define the term power set. (1 marks)

ii. Let A={1,3,{1,3}}. Find the power set of A. (2 marks)

b) Let A ={1,2,3} and B={1,2,3,4}. Let $f: A \rightarrow B$ be defined by f (x)= x+1. Determine if the function f is surjective or injective. (3 marks)

c) Proof by mathematics induction that

$$1\bullet 2 + 2\bullet 3 + 3\bullet 4 + \ldots + n(n+1) = \frac{n(n+1)(n+2)}{3}$$ (5 marks)

d) Negate each of the following statements:

i. The exists an integer that is not divisible by 5. (1 marks)

ii. John is at the library or he is not at the book store. (1 marks)

e) Proof that a statement is logically equivalent to its contrapositive. (4 marks)

f) Draw the truth table for $q \rightarrow (r \vee (\sim s))$. (4 marks)

g) Determine the validity of the following argument. (4 marks)

$$\begin{array}{c} \sim p \Rightarrow q \\ w \wedge p \\ q \\ \hline p \Rightarrow q \end{array}$$

h) Show by shading the region described by the following $(A \setminus B)^c \cap (B \cup C)^c$ (3 marks)

i) State any four methods of mathematical proofs. (2 marks)

<u>Question Two (20 Marks)</u>

a) Define the following terms "Mapping" (2 marks)

b) Let f be function, $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(a) = a + 1$ whereas $g : \mathbb{R} \rightarrow \mathbb{R}$ is a function defined by $g(b) = \begin{cases} b, & b < 5 \\ b - 1 & b \geq 5 \end{cases}$. Compute

i. f^{-1} (1 marks)

ii. $f^{-1}(-2)$ (1 marks)

iii. $g_o f$ (2 marks)

iv. $g_o f$ (3) (1 marks)

v. $g_o f_o f_o g_o f$ (2) (3 marks)

c) Determine the validity of the following argument.

If Akinyi bought a house, then either she sold her car or she borrowed money from Muiganania sacco. Akinyi did not sell her car or she did not buy a house. Akinyi has not borrowed money from Muiganania sacco. Therefore if Akinyi did not sell her car then she did not buy a house. (10 marks)

Question Three (20 Marks)

a)

i. Define the term "Cartesian product" of two sets A and B. (2 marks)

ii. Find A x B and B x A given that A={a, b} and B={a, c, b} (3 marks)

b) State any three De-Morgan's law of sets. (3 marks)

c) Out of 300 students taking discrete mathematics, 60 take coffee, 27 take cocoa, 36 take tea, 17 take tea only, 47 take chocolate only, 7 take chocolate and cocoa, 3 take chocolate, tea and cocoa, 20 take cocoa only, 2 take tea, coffee and chocolate, 30 take coffee only, 9 take tea and chocolate whereas 12 take tea and coffee.

i. Express this information on a Venn diagram. (6 marks)

ii. Find how many take any beverage. (2 marks)

iii. Find how many take cocoa and tea (2 marks)

iv. Find how many take at least two beverages (2 marks)

Question Four (20 Marks)

a) Use mathematical induction to proof that

$$\sum_{k=1}^{n} \frac{1}{(2k-1)(2k+1)} = \frac{n}{2n+1}$$ (8 marks)

b) Proof that the

i. Product of any two rational numbers is rational. (4 marks)

ii. Product of any two irrational numbers is not always irrational. (2 marks)

c) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (4 marks)

d) List all positive prime numbers less than 30 (2 marks)