MURANG'A UNIVERSITY COLLEGE
(A constituent college of Jomo Kenyatta University of Agriculture and Technology)

ICS 2211: NUMERICAL LINEAR ALGEBRA

MAIN EXAMINATION

DATE: 10 DECEMBER 2013 **TIME 2HOURS**

SECTION I (Compulsory)

QUESTION ONE (30 MARKS)

a. Let A be the 3×3 matrix $\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Find $M_{23}$and A_{23} (2 marks).

b. Find the determinant of $B = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 4 & 1 & -1 \end{pmatrix}$ (6 marks)

c. Solve the following linear system using Gaussian Elimination (6 marks)

$$2x + y + 2z = 5$$
$$-2x + 2z = 2$$
$$-2x + y + z = 0$$

d. Find the eigen values and eigen vectors for $C = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$. (6 marks)

e. Check that the coefficient matrix of the following system is symmetric

f. and positive definite. Then, solve the system using LU factorization. (10 marks)

$$X_1 - 2x_2 + 3x_3 = 2$$
$$2x_1 - 3x_2 + 2x_3 = 9$$
$$3x_1 + x_2 - x_3 = -1$$

SECTION II

Instructions: Answer any TWO questions

QUESTION TWO (20 marks)

a. Compute the LU factorization with partial pivoting, **PA** = **LU**, for the following matrix (7 marks)

$$A = \begin{bmatrix} 1 & 2 & -4 \\ 2 & 2 & 0 \\ 1 & 3 & 4 \end{bmatrix}$$

b. i. Compute the inverse of $D = \begin{pmatrix} 1 & 2 & 4 \\ 2 & 3 & 4 \\ 2 & 5 & 6 \end{pmatrix}$. (7 marks)

ii. Use the inverse to solve the System Dx = b where $b = (-1 \quad 1 \quad 1)^T$ (3 marks)

c. Let **A** and **B** be two nonsingular lower triangular $m \times m$ matrices. Show that the product **AB** is also lower triangular. (3 marks)

QUESTION THREE (20 MARKS)

a. Let A be a nonsingular matrix.

i. Show that A^{1} is unique. (3 marks)

ii. Show that A^{1} is nonsingular and $(A^{1})^{1} = A$. (3 marks)

iii. Show that A^T is nonsingular and $(A^T)^{1} = (A^{1})^T$. (3 marks)

iv. If B is nonsingular, show that AB is nonsingular and $(AB)^{1} = B^{1}A^{1}$. (3 marks)

b. Let $E = \begin{pmatrix} -3 & 1 & 2 \\ -2 & 0 & 2 \\ -2 & 1 & 1 \end{pmatrix}$. Find the Eigen values and eigen vectors for E. (8 marks)

QUESTION FOUR (20 MARKS)

a. Solve the following linear system using Cramer's rule (10 marks)

$$2x + 8y + 3z = 2$$
$$x + 3y + 2z = 5$$
$$2x + 7y + 4z = 8$$

b. Solve the following system of equations by PA= LU factorization:

$$x_1 + 2x_2 + 4x_3 = 1$$
$$4x_1 + 5x_2 + 6x_3 = 2$$
$$7x_1 + 8x_2 + 9x_3 = 3$$

(10 marks)