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University Examinations 2014/2015

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF TECHNOLOGY IN MECHANICAL ENGINEERING, ELECTRICAL AND ELECTRONIC ENGINEERING AND CIVIL ENGINEERING.

SME 3111: MATHEMATICS I

DATE: DECEMBER 2014 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE (30 MARKS)

a) Identify the property of real numbers being applied in each of the following (2 marks)

(i) $(4+5)+6=6+(4+5)$

(ii) $(5+3)+2=5+(3+2)$

b) Find real numbers x and y such that $3x+2iy-ix+5y=7+5i$ (3 marks)

c) Find the general and normal equations to the line through point (2,5) and (3,7) (4 marks)

d) Evaluate the following limit (3 marks)

$$\lim_{x\to 2}\left[\frac{\sqrt{x^2+5}-3}{x^2-2x}\right]$$

e) Find $f^1(x)$ from the first principles given that $f(x)=\sqrt{x-1}$. State the domain of $f^1(x)$. (4 marks)

f) Find the derivatives of the following functions.

(i) $y=(4x+\frac{1}{2})(x^2+1)$ (1 mark)

(ii) $y = \frac{x^3}{2x+1}$ (1 mark)

(iii) $(3x - 2x^2)^3$ (1 mark)

g) Find $\frac{dy}{dx}$ if $y^2 + y^3 = 4x^3$ (3 marks)

h) Find the maximum and minimum values of the curve $y = 2x^3 - 6x + 5$ (4 marks)

i) The time of a swing T of a pendulum is given by T=k$\sqrt{l}$, where k is a constant. Determine the percentage change in the time of the swing if the length of the pendulum l changes from 32.1cm to 32.0cm. (4 marks)

QUESTION TWO (20 MARKS)

a) Find the points on the curve $y = x^4 - 6x^2 + 4$ where the tangent line is horizontal. (4 marks)

b) Evaluate $\lim_{n \to 0} \frac{3(x+n)^2 - 3x^2}{n}$ (3 marks)

c) Find $\frac{dy}{dx}$ if

(i) $y = x^2e^x - 2xe^x + 2e^x$ (3 marks)

(ii) $y = ln\left[\frac{x^2 - 2}{2x^2 - 4}\right]$ (3 marks)

(iii) $y = 6\sqrt{x} + 5\cos x$ (2 marks)

d) A water tank has the shape of an inverted circular cone with base radius 2m and height 4m. If water is being pumped into the tank at a rate of 2m^3/min, find the rate at which the water level is rising when the water is 3m deep (5 marks)

QUESTION THREE (20 MARKS)

a) Find the value of k for which f(x) is continuous. (4 marks)

$$f(x) \quad \begin{cases} x+k, if\ x<0 \\ -2+k^2, if\ x \geq 0 \end{cases}$$

b) Given y= $\tan^{-1} x$, find $\frac{dy}{dx}$ in terms of x only (4 marks)

c) Find the equation of the normal to the curve $y = x^2 - x - 2$ at the point (1,2).

(3 marks)

d) A farmer has 2400m of fencing wire and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area? (4 marks)

e) Determine where the curve x^3-3x+1 is concave upward and where it is concave downward. Find the inflection points and sketch the curve. (5 marks)

QUESTION FOUR (20 MARKS)

a) Change the equation $r = 1 + cos\ \theta$ into Cartesian coordinates (3 marks)

b) Find the focus, the equation of the directrix, the length of the latus rectum for the parabola $5y^2$=24x (4 marks)

c) (i) Express the following complex number in polar form -5+5i (3 marks)

(iii) Solve the quadratic equation, writing the real and the imaginary parts. $3x^2$-4x+2=0

(3 marks)

d) Determine the height and radius of a cylinder of volume 200cm^3 which has the least surface area. (7 marks)

QUESTION FIVE (20 MARKS)

a) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point (3,4) (3 marks)

b) Find the gradient function of $f(x) = x^3 - x$ from the first principles (4 marks)

c) The functions f and g are defined by $f(x) = 7x + 1$ and $g(x) = \frac{x}{3} - 1$. Find $(f_0 g)^{-1}(x)$. (4 marks)

d) Differentiate the function $f(x) = \ln\left(e^{x^2}\right)$ (3 marks)

e) A rectangular sheet of metal having dimensions 20cm by 12cm has squares removed from each of the four corners and the sides bent upwards to form an open box. Determine the maximum possible volume of the box. (6 marks)