

MURANG'A UNIVERSITY COLLEGE

A Constituent College of Jomo Kenyatta University of Agriculture and Technology

University Examination 2015/2016

YEAR I SEMESTER I EXAMINATION FOR THE DEGREE OF BACHELOR OF

SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMM 2101: FOUNDATION MATHEMATICS – I

DATE: December 2015

TIME: 2 Hours

Instructions: Attempt question <u>One</u> and <u>Two</u> other questions

Question One (30 Marks)

- a) Given the sets $A = \{2,5,8,9,12,14\}$ and $B = \{2x | 3 < x < 7, x \in \mathbb{N}\}$ find the symmetric difference of the sets A and B (4 Marks)
- b) State the contra positive of the statement 'If someone has read "the river between", then he remembers the character of Waiyaki' (3 Marks)
- c) Let *p*, *q* and *r* be three statements. Construct the complete truth table for the logical statement $\neg(p \land q) \land (\neg r)$ (5 Marks)
- d) Given the function f(x) = 3x + 2, $g(x) = \frac{1}{x+3}$ and h(x) = x 5, show that $[(f \circ g) \circ h](x) = \frac{2x-1}{x-2}$ (5 Marks)
- e) A committee of 7 persons is to be selected from 6 women and 5 men. In how many ways can this be done if the committee is to have more men than women? (3 Marks)
- f) Given the complex numbers $z_1 = 3 5i$, $z_2 = 1 + 2i$ and $z_3 = 4 + i$, express $\frac{2z_1 + z_2}{z_3}$ in Cartesian form (5 Marks)
- g) Prove that $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \left(\frac{x+y}{1-xy}\right)$ (5 Marks)

Question Two (20 Marks)

- a) Find the principal value of ln(3 + 4i) (4 Marks)
- b) Find all the cube-roots of $1 i\sqrt{3}$ (10 Marks)
- c) Express $Sin^5\theta$ into sines of multiple angles (6 Marks)

Question Three (20 Marks)

a) Let $a, b, c \in \mathbb{Z}$ prove that if a|b and b|c then a|c (4 Marks)

- b) Given that $a \equiv b \pmod{n}$ where $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, prove that $a^2 \equiv b^2 \pmod{n}$ (6 Marks)
- c) Prove that if x is an even integer, then $x^2 6x + 5$ is odd (4 Marks)
- d) Use the principle of mathematical induction to prove that

$$1^{2} + 2^{2} + 3^{2} + \dots + n^{2} = \frac{1}{6}n(n+1)(2n+1)$$
 (6 Marks)

Question Four (20 Marks)

- a) Define the following terms
 - (i) Injective function
 - (ii) Surjective function
 - (iii) Bijective function (3 Marks)
- b) Let $A = \{3, 4, 5, 7, 8\}$ and define $f: A \to \mathbb{R}$ by $f(x) = x^2 + 1$. Determine the range of the function f(x) (3 Marks)
- c) Given the sets $A = \left\{ x \mid \frac{x}{2} \in \mathbb{N}, x < 11 \right\}$ and $C = \{y \mid y \in \mathbb{N}, y \le 10\}$, find another set B such that the sets A and B are partitions of the set C (4 Marks)
- d) A Market research survey on the reading habits of 300 persons gave the following data in respect to three leading newspapers (Daily Nation, The standard and The People daily)

Newspaper	Daily Nation	The Standard	The People Daily	Daily Nation and Standard	Standard and People Daily	Daily Nation and People Daily	All the three Newspapers
Number of persons	100	140	125	50	40	30	20

Represent the information in a Venn diagram hence determine (7 Marks)

- (i) The number of persons who read exactly one newspaper. (1 Mark)
- (ii) The number of persons who read exactly 2 newspapers. (1 Mark)
- (iii) The number of persons who do not read newspapers (1 Mark)

Question Five (20 Marks)

- a) Prove that $Sin \theta Cos 2\theta Sin \theta = -2Sin^3\theta$ (3 Marks)
- b) Find the values of the constants *R* and α such that $5 \sin \theta + 8 \cos \theta = R \cos(\theta + \alpha)$ hence solve the equation $5 \sin \theta + 8 \cos \theta = 7 \text{ for } 0 \le \theta \le 360^{\circ}$ (9 Marks)

c) Solve the equation
$${}^{x}C_{4} = 5 \left\{ {}^{(x-2)}C_{3} \right\}$$
 (8 Marks)