MURANG'A UNIVERSITY COLLEGE
A Constituent College of Jomo Kenyatta University of Agriculture and Technology
University Examination 2015/2016
YEAR I SEMESTER I EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN APPLIED STATISTICS WITH PROGRAMMING

AMM 2101: FOUNDATION MATHEMATICS - I
DATE: December 2015
TIME: 2 Hours
Instructions: Attempt question One and Two other questions

## Question One (30 Marks)

a) Given the sets $A=\{2,5,8,9,12,14\}$ and $B=\{2 x \mid 3<x<7, x \in \mathbb{N}\}$ find the symmetric difference of the sets A and B
b) State the contra positive of the statement 'If someone has read "the river between", then he remembers the character of Waiyaki'
c) Let $p, q$ and $r$ be three statements. Construct the complete truth table for the logical statement $\neg(p \wedge q) \wedge(\neg r)$
d) Given the function $f(x)=3 x+2, g(x)=\frac{1}{x+3}$ and $h(x)=x-5$, show that

$$
\begin{equation*}
[(f \circ g) \circ h](x)=\frac{2 x-1}{x-2} \tag{5Marks}
\end{equation*}
$$

e) A committee of 7 persons is to be selected from 6 women and 5 men. In how many ways can this be done if the committee is to have more men than women?
(3 Marks)
f) Given the complex numbers $z_{1}=3-5 i, z_{2}=1+2 i$ and $z_{3}=4+i$, express $\frac{2 z_{1}+z_{2}}{z_{3}}$ in Cartesian form
g) Prove that $\tan ^{-1} x+\tan ^{-1} y=\tan ^{-1}\left(\frac{x+y}{1-x y}\right)$

## Question Two (20 Marks)

a) Find the principal value of $\ln (3+4 i)$
b) Find all the cube-roots of $1-i \sqrt{3}$
c) Express $\operatorname{Sin}^{5} \theta$ into sines of multiple angles

## Question Three (20 Marks)

a) Let $a, b, c \in \mathbb{Z}$ prove that if $a \mid b$ and $b \mid c$ then $a \mid c$
b) Given that $a \equiv b(\bmod n)$ where $a, b \in \mathbb{Z}$ and $n \in \mathbb{N}$, prove that $a^{2} \equiv b^{2}(\bmod n)$
c) Prove that if $x$ is an even integer, then $x^{2}-6 x+5$ is odd
d) Use the principle of mathematical induction to prove that

$$
\begin{equation*}
1^{2}+2^{2}+3^{2}+\cdots+n^{2}=\frac{1}{6} n(n+1)(2 n+1) \tag{6Marks}
\end{equation*}
$$

## Question Four (20 Marks)

a) Define the following terms
(i) Injective function
(ii) Surjective function
(iii) Bijective function
(3 Marks)
b) Let $A=\{3,4,5,7,8\}$ and define $f: A \rightarrow \mathbb{R}$ by $f(x)=x^{2}+1$. Determine the range of the function $f(x)$
c) Given the sets $A=\left\{x \left\lvert\, \frac{x}{2} \in \mathbb{N}\right., x<11\right\}$ and $C=\{y \mid y \in \mathbb{N}, y \leq 10\}$, find another set B such that the sets $A$ and $B$ are partitions of the set $C$
(4 Marks)
d) A Market research survey on the reading habits of 300 persons gave the following data in respect to three leading newspapers (Daily Nation, The standard and The People daily)

| Newspaper | Daily <br> Nation | The <br> Standard | The People <br> Daily | Daily <br> Nation and <br> Standard | Standard and <br> People Daily | Daily Nation <br> and People <br> Daily | All the three <br> Newspapers |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of <br> persons | 100 | 140 | 125 | 50 | 40 | 30 | 20 |

Represent the information in a Venn diagram hence determine
(7 Marks)
(i) The number of persons who read exactly one newspaper.
(ii) The number of persons who read exactly 2 newspapers.
(iii) The number of persons who do not read newspapers

## Question Five (20 Marks)

a) Prove that $\operatorname{Sin} \theta \operatorname{Cos} 2 \theta-\operatorname{Sin} \theta=-2 \operatorname{Sin}^{3} \theta$
b) Find the values of the constants $R$ and $\alpha$ such that $5 \operatorname{Sin} \theta+8 \operatorname{Cos} \theta=R \cos (\theta+\alpha)$ hence solve the equation $5 \operatorname{Sin} \theta+8 \operatorname{Cos} \theta=7$ for $0 \leq \theta \leq 360^{\circ}$
c) Solve the equation ${ }^{x} C_{4}=5\left\{{ }^{(x-2)} C_{3}\right\}$

