MURANG'A UNIVERSITY COLLEGE

A Constituent College of Jomo Kenyatta University of Agriculture and Technology

University Examination 2015/2016

YEAR II SEMESTER I EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY

ICS 2211: NUMERICAL LINEAR ALGEBRA

DATE: 9TH December 2015 **TIME: 2 HOURS**

Instructions: Attempt question **One** and **Two** other questions

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Question One (30 Marks)

a) Given the matrices

$$A = \begin{pmatrix} 3 & 6 \\ 5 & 1 \\ 4 & 0 \end{pmatrix} \quad and \quad B = \begin{pmatrix} 3 & -2 & 7 \\ 5 & 8 & 3 \end{pmatrix}$$

Show that $A \cdot B$ is singular (4 Marks)

b) Prove that the inverse of a square matrix is unique. (6 Marks)

c) Use the Chio's condensation method to find the determinant of the matrix

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & -2 & -3 \\ -3 & 6 & 5 \end{pmatrix}$$ (5Marks)

d) Use the iterative method to find the inverse of the matrix $A = \begin{pmatrix} 4 & 5 \\ 1 & -2 \end{pmatrix}$ given that its approximate inverse is $\begin{pmatrix} 0.1 & 0.4 \\ 0.1 & -0.3 \end{pmatrix}$ *(Perform two iterations)* (5Marks)

e) Show that the characteristic equation of any 2 by 2 matrix A is given by

$\lambda^2 - [tr(A)]\,\lambda + |A| = 0$

$where \quad tr(A) - \text{ is the trace of the matrix } A \text{ and}$

$|A| - \text{is the determinant}$ (5 Marks)

f) Find a square matrix of order 2 whose Eigen values are $\lambda_1 = -1,\ \lambda_2 = 2$ and whose Eigen vectors are $X_1 = (2\ \ 3)^T\ and\ \ X_2 = (3\ \ 4)^T$ (5 Marks)

Question Two (20 Marks)

a) Find the Eigen values and the corresponding Eigen vectors of the matrix

$$A = \begin{pmatrix} 1 & 3 & 0 \\ 1 & 2 & 1 \\ -2 & 1 & -1 \end{pmatrix}$$ (13 Marks)

b) Use power method to find the dominant Eigen value of the matrix

$$A = \begin{pmatrix} 5 & 2 & 4 \\ -3 & 6 & 2 \\ 3 & -3 & 1 \end{pmatrix}$$

Use $X^{(0)} = (1 \quad 0 \quad 0)^t$ and perform two iterations (7 Marks)

Question Three (20 Marks)

a) Given the matrix $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$ find a matrix P such that $P^{-1}AP = D$ where D is a diagonal matrix. (9Marks)

b) Use matrix inversion method to find the solution to the linear system of equations;

$$x_1 - 4x_2 - 2x_2 = 21$$

$$2x_1 + x_2 + 2x_3 = 3$$

$$3x_1 + 2x_2 - x_3 = -2$$ (11Marks)

Question Four (20 Marks)

a) Use Cramer's rule to find the solution to the linear system of equations

$$4x - 5y - z = 9$$
$$3x - 2y + 2z = 16$$
$$x + 3y + 4z = 15$$ (11 Marks)

b) Apply the method of steepest descent to find the local minimum of the function

$f(x) = x_1^2 + x_2^2$ given$X^{(0)} = (2,2)$ (9 Marks)

Question Five (20 Marks)

a) Find the spectral norm ($\|A\|_2$) of the matrix

$$A = \begin{pmatrix} 1 & -1 \\ -1 & 3 \end{pmatrix}$$ (4 Marks)

b) Consider the matrix

$$A = \begin{pmatrix} 13 & 5 \\ 2 & 4 \end{pmatrix}$$

Given that the dominant Eigen value is $\lambda = 14$ and the associated eigenvector is $X = \begin{pmatrix} 1 \\ 0.2 \end{pmatrix}$. Use deflation technique to find the other Eigen value of the matrix A and the associated Eigen vector (*Perform two iterations)* (7 Marks)

c) Set-up the Gauss-Seidel method for the system of equations

$8x_1 - x_2 + x_3 = 5$

$x_1 - 5x_2 - 2x_3 = -14$

$3x_1 - 2x_2 + 10x_3 = 10$

Hence solve the system taking $x^{(0)} = (1 \quad 1 \quad 1)^t$(perform three iterations) (9 Marks)