

**W1-2-60-1-6**

**JOMO KENYATTA UNIVERSITY**

**OF**

**AGRICULTURE AND TECHNOLOGY**

**UNIVERSITY EXAMINATIONS 2014/2015**

**YEAR 4 SEMESTER II EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE**

**STA 2493: SURVIVAL ANALYSIS**

**DATE: APRIL 2015 TIME: 2 HOURS**

**INSTRUCTIONS: Answer Question One and Any Other Two Questions**

**QUESTION ONE (30 MARKS)- Compulsory**

1. i. Define survival analysis (2marks)

ii. List any four fields in which survival analysis can be applied (4marks)

1. i. Given that the hazard function is

 $h\left(t\right)=\sqrt[a]{t}$ Where $a>0$

 Calculate the survival function and the corresponding density function (4marks)

ii. Let $F\_{0}\left(t\right)=1-\left[1-\frac{t}{120}\right]^{\frac{1}{6}}$

 Calculate the probability that;

1. A new born life survives beyond age 30 (2marks)
2. A life age 30 dies before age 50 (2marks)
3. A life age 60 survives beyond age 65 (2marks)
4. Calculate the limiting age hence discuss its implication under this model. (2marks)
5. i. Calculate the Kaplan-Meier estimate of the survival function $s(t)$ of the following data. $δ\_{i}=1$ if individual $\acute{i}$ died at time $t\_{\acute{i}}$ and $δ=0$ if $\acute{i}$ was censored at that time. $\acute{i}=1,………,8$

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| $$\acute{i}$$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $$t\_{\acute{i}}$$ | 2 | 5 | 8 | 11 | 12 | 15 | 20 | 23 |
| $$δ\_{\acute{i}}$$ | 1 | 1 | 1 | 0 | 0 | 1 | 1 | 0 |

ii. Hence sketch the survival function estimate (6marks)

1. Consider a study carried out to evaluate the effect of a drug for high blood pressure. Elders’ volunteers are randomly allocated to one of the three treatments; a placebo, the drug that is currently most common and the new drug that has been developed. The study last for ten years and the event of interest is occurrence of any kind of heart disease including death resulting from heart disease. Briefly discuss three situations from this study that might lead to an observation that is right censored. (6marks)

**QUESTION TWO (20 marks)**

1. Show that if the hazard function is given by

$λ\left(t\right)=αβ\left(αt\right)^{β-1}exp⁡\{\left(αt\right)^{β}\}$ Then the survival function will be

$s\left(t\right)=exp⁡\{-[exp\left(αt\right)^{β}-1\}$ (5marks)

1. A doctor treated 10 patients and then tested them each day until the test was negative. The number of days for testing (T) was a random variable shown in the table below

Days (T) 1 2 3 4 5 6

No. of patients tested that day 10 10 8 5 2 0

1. Determine $s(t)$
2. Calculate $≡(T)$ (6marks)
3. If survival times in the absence of censoring are distributed according to a Weibull distribution with parameters k and λ, the hazard and survival functions can be written as

$h\left(t\right)=λkt^{k-1}$

$s\left(t\right)=exp⁡(-λt^{k})$

If we have observed data of the form $\left(t\_{i},δ\_{i}\right)$ where $δ\_{i}$=1 if individual i fails at time $t\_{\acute{i}}$ and $δ\_{\acute{i}}$ =0 if $\acute{i}$ is right censored at $t\_{\acute{i}}$ for $\acute{i}$=$1,……,M$

1. Compute the log-likelihood function
2. Describe briefly how you might find the maximum-likelihood estimates of $k$ and $λ$ (9marks)

**QUESTION THREE (20 marks)**

1. Let T be a continuous random variable with density given by

$f\left(t\right)=\left\{\begin{matrix}λe^{-λt},t>0\\0 elsewhere\end{matrix}\right.$

1. Find the expression for the Pth quantile (4marks)
2. Hence or otherwise determine the median life time in terms of λ (4marks)
3. In a particular actuarial survival model, we have

$μ\_{x}=\frac{0.01}{1-0.01x}$ for $0\leq x<100$

**QUESTION FOUR (20 marks)**