EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

FIRST SEMESTER EXAMINATIONS

FIRST YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS)

TBS 104: INTRODUCTION TO MATHEMATICS FOR ECONOMISTS

DATE: DECEMBER 10, 2015 **TIME: 14:00-16:00**

INSTRUCTIONS:
Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Let $A = \{a, c, g, i\}$, $B = \{b, d, e, f, i\}$, and $C = \{a, b, c, d, e\}$ be subsets of a universal set $\mathcal{E} = \{a, b, c, d, e, f, g, h, i\}$. Determine the following:

(i) $A \cup C$ (1 mark)

(ii) $A \cap B$ (1 mark)

(iii) $n(A \cup B)$ (2 marks)

(iv) $B - C$ (2 marks)

(v) $A \Delta C$ (2 marks)

b) Determine the domain of the function $f(x) = \frac{\sqrt{x^2-x-2}}{x+4}$ (4 marks)

c) If $g(x) = 2x^2 + 4$ and $h(x) = \frac{5}{x}$, determine the composite function $h \circ g$ (2 marks)

d) Determine $\frac{dy}{dx}$ in each of the following:

(i) $y = \sqrt[5]{x} - \frac{4}{x^2}$ (2 marks)

(ii) $y = \sqrt{3x^2 - 5x + 9}$ (2 marks)

e) Evaluate the integral $\int \left(\sqrt{x} + 5x^2 - \frac{3}{x^2}\right) dx$ (2 marks)

f) Compute the determinant of the matrix $M = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 3 & -1 & 2 \end{bmatrix}$. (2 marks)

g) Determine the production level that will maximize profit for a company with cost function $C(x) = 16000 + 500x - 1.6x^2 + 0.004x^3$ and demand function $p(x) = 1700 - 7x$

(8 marks)

QUESTION TWO

(a) Determine $\frac{dy}{dx}$ in each of the following:

(i) $y = \sqrt[5]{x} + \frac{3}{x^2} + 7\cos\theta$ (2 marks)

(ii) $y = \frac{5+2x}{x+3}$ (2 marks)

(iii) $y = \sqrt[3]{9x^2 + 15x - 17}$ (3 marks)

(iv) $y = (2x + 1)\sqrt{3x}$ (2 marks)

(b) In a certain company, the cost (in shillings) of producing x items is given by $C(x) = 25000 + 120x + 0.1x^2$. Determine:

(i) the cost of producing 1000 items (2 marks)

(ii) the average cost of producing 1000 items (2 marks)

(iii) the marginal cost at a production level of 1000 items (2 marks)

(iv) the production level that will minimize the average cost (3 marks)

(v) the minimum average cost (2 marks)

QUESTION THREE

(a) Prove that if A and B are any two subsets of a universal set Ɛ, then

$(A \cap B)^c = A^C \cup B^C$ (6 marks)

(b) Use the subsets $A = \{a, c, g, i\}$ and $B = \{b, d, e, f, i\}$ and the universal set $\mathcal{E} = \{a, b, c, d, e, f, g, h, i\}$ to illustrate that $(A \cap B)^c = A^C \cup B^C$. (6 marks)

(c) Solve for x in $2\log_3(x + 1) - 2 = 2\log_3 x$ (8 marks)

QUESTION FOUR

a) Evaluate the following integrals

(i) $\int \frac{dx}{\sqrt{2x+1}}$ (5 marks)

(ii) $\int xe^x \, dx$ (4 marks)

b) Solve the following system of linear equations using the inverse matrix method:

$$2x - 3y + z = -1$$
$$x - y + z = 0$$
$$3x + 2y + 5z = 11$$

(11 marks)

QUESTION FIVE

a) Define the following terms as used in Economics:

(i) Consumer's surplus (2 marks)

(ii) Producer's surplus (2 marks)

b) If the demand and supply functions under pure competition are given by $p_d(x) = 16 - x^2$ and $p_s(x) = 2x^2 + 4$, respectively, determine the consumer's surplus and producer's surplus at the market equilibrium price. (16 marks)

---END---