EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SPH 309: QUANTUM MECHANICS I

DATE: APRIL 12, 2016 **TIME: 08:30-10:30**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions

QUESTION ONE

a) State the De Broglie's' hypothesis and write the equation. (2 Marks)

b) Briefly give two formulations of quantum mechanics. (4 Marks)

c) What is perturbation? Hence differentiate between time dependent and time – independent perturbation. (6 Marks)

d) Briefly define the following terms i) Tunneling.

ii) Normalization

iii) Potential barrier

iv) Variational method. (8 Marks)

e) State the uncertainty principle and write the two formulae's for it in terms of momentum and energy. (4 Marks)

f) What is Born approximation? (2 Marks)

g) Differentiate between linear operators and observables giving an example of each. (4 Marks)

QUESTION TWO

a) Use the variational principle to estimate the ground state energy of the anharmonic oscillator, *H* = ^*p*2=2*m* + ¸*x*4, and com-pare with exact result *E*0 =1:06¸1=3(¹*h*2=2*m*)2=3. (10 Marks)

b) A particle in an in¯nite square well (of width *a*) has as its initial wave function

   a. an equal mixture of the ¯rst two stationary states:

   ª(*x*; 0) = *C*[*Ã*1(*x*) + *Ã*2(*x*)]

   i) Normalise ª(*x*; 0). (That is, ¯nd*C*.)

   ii) Find ª(*x*; *t*) and *j*ª(*x*; *t*)*j*2. Express the latter in terms of sin and cosusing*eiµ* = cos*µ* + *i* sin *µ*. Use *!*= ¼2¹*h*=2*ma*2.

   iii) Compute < *x* >. Notice that it oscillates in time. What is the frequencyof the oscillation? What is the amplitude?

   iv) Compute < *p* >.

   v) Find the expectation value of the Hamiltonian operator, *H*, in terms of *E*1and *E*2.

(12 Marks)

QUESTION THREE

a) Highlight and give a brief description of thye key features of Stern-Gerlach experiment. (6 Marks)

b) A Stern-Gerlach apparatus is aligned along the *z*direction, and a second one is aligned at 45± with respect to this in the *z ¡ x*plane. A neutral spin-1/2 particle is prepared with spin *k* ^*z* and then passed through both in succession. (i) What is the probability a detector located after the two SG setups ¯ndsthe particle to have spin up (*k* ^*z*)? (ii) What is the probability a detector measures the spin to be along the ^*y*-direction? (14 Marks)

QUESTION FOUR

a) Write the expression *hÃjÃi*= 1 as an explicit integral equation in three dimensions, assuming that *jÃi* represents a wave function *Ã*(~*r*). Supposeyou have *jÃi*=P*n cnjni*where the *jni*are a complete set of orthonormal states. What conditions does the above equation impose on the *cn*? (3 Marks)

b) How many degenerate levels are there for a hydrogen atom with principal quantum number *n*? Are any of these degeneracies lifted by the spin-orbit interaction? Justify your answer. (5 Marks)

c) What is the big deference between a particle with spin quantum numbers *s* = 1=2 and one with *s* = 1? (4 Marks)

d) Suppose that the operator corresponding to some observable is called *Q*.List 2 properties of this operator and/or of its eigenfunctions *jni*. Thelatter satisfy *Qjni*= *qnjni*. Suppose further that the quantum-mechanical state of a system is given by *jÃi*=P*n cnjni*with several of the *cn 6*= 0. Ifyou were to make a *single measurement* of the observable *Q*, what would you get as a result? (4 Marks)

e) Two quantum mechanical particles have orbital angular momentum *l* = 1and spin angular momentum *s* = 0. Suppose that there is some coupling of the two particles. List the values that the total angular momentum *j*of the two-particle system may take on. For each *j*, state what are the possible values for the *z* component. (4 Marks)

QUESTION FIVE

Consider an in¯nite well for which the bottom is not °at, as sketched here. If the slope is small, the potential *V* = ²*jxj*=*a* may be considered as a per- turbation on the square-well potential

over¡*a*=2 · *x* · *a*=2.

a) Calculate the ground-state energy, correct to ¯rst order in perturbation theory.
b) Calculate the energy of the ¯rst excited state, correct to ¯rst order in perturbation theory.
c) Calculate the wave function in the ground state, correct to ¯rst order in perturbation theory.
d) At what value of ² does perturbation theory break down? Justify your answer.

(20 Marks)

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