EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE (STATISTICS)

STA 122: COMPUTATIONAL METHODS AND DATA ANALYSIS I

DATE: APRIL 7, 2016 **TIME: 11:00AM-01:00PM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions

QUESTION ONE

a) i) State two formats used to store numbers in a computer. (2 Marks)

ii) Given that $\pi = \frac{22}{7}$ and that the true value of $\pi = 3.14159265$, Calculate the relative error encountered while using $\pi = \frac{22}{7}$. (3 Marks)

b) Convert the following numbers to the indicated form;

i) 12570_8 . Octal to decimal number. (2Marks)

ii) $19FDE_{16}$. Hexadecimal to decimal number. (3Marks)

iii) 12.45_{10}. Decimal to binary number. (4 Marks)

c) If $f(x)$ tends to a limit l as $x \to a$, then for a number ε (however small), it must be possible to find a number η such that $|f(x)-l|<\varepsilon \text{ where } |x-a|<\eta$. Consider $f(x)=1-\frac{1}{x+2}$. Given that $\lim_{x\to 1}=\frac{2}{3}$, determine the value of η.

(5 Marks)

d) Show that the Taylor's series centered at $x=0$ converge to $\sin x$ for all values of x.

(3 Marks)

e) Given the ideal gas law $PV = nRT$ where R is a constant for all gases $R = 8.3143+\varepsilon, |\varepsilon| = 0.0012$ determine the error in T.

(5 Marks)

f) Determine the relative error in $\frac{x_A}{y_A}$ compared to $\frac{x_T}{y_T}$ given that $x_T = x_A + \varepsilon$ and $y_T = y_A + \eta$

(3 Marks)

QUESTION TWO

a) Consider IEEE double precision floating point arithmetic using round to the nearest. Let a, b and c be normalized double precision floating point numbers and let $\oplus, \otimes \text{ and } \phi$ denote correctly rounded floating point addition, multiplication and division.

i) Is it necessarily true that $a \oplus b = b \oplus a$? Explain why and give an example.

(1 Mark)

ii) Is it necessarily true that $(a \oplus b) \oplus c = a \oplus (b \oplus c)$? Explain and give an example.

(4 Marks)

iii) I) Determine the maximum possible relative error in the computation $(a \otimes b)\phi c$ assuming $c \neq 0$.

(4 Marks)

II) Suppose $c = 0$, what are the possible values that $(a \otimes b)\phi c$ could be Assigned.

(3 Marks)

b) One way to approximate the derivative of a function f is

$$f'(x) = \frac{f(x+h) - f(x-h)}{2h}$$

for some small number h (centered different formula). Assuming that $f \in c^2$

Use Taylor's theorem to determine the accuracy of this approximation

(5 Marks)

c) Show that with this formula, we can approximate a derivative to abort $2/3$ power of the machine precision

(3 Marks)

QUESTION THREE

Consider the linear system

$$\begin{aligned} 9x_1 + x_2 + x_3 &= b_1 \\ 2x_1 + 10x_2 + 3x_3 &= b_2 \\ 3x_1 + 4x_2 + 11x_3 &= b_3 \end{aligned}$$

a) Solve for x_k through

i) Jacobi iteration
(9 Marks)

ii) Gauss- seidel iteration (10 Marks)

b) Comment on the rate of convergence between the two iteration methods. (1 Mark)

QUESTION FOUR

a) Use Taylor's series expression

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + f''(x_0)(x - x_0) + f'''(x_0)(x - x_0) + \ldots$$

With $n = 0$ *to* 6 to approximate $f(x) = \cos(x)$ at $x = \frac{\pi}{3}$ on the basis of $f(x)$ and its derivatives at $x_0 = \frac{\pi}{4}$. (5 Marks)

b) After each new term is added , compute the true percent relative error ε_t (11 Marks)

c) What value of n is required for the absolute value of the percent error $|\varepsilon|$ to fall below a pre-specified error criterion ε_s conforming to six (6) significant figures?

(4 Marks)

QUESTION FIVE

Suppose that you are given a polynomial

$P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$ Of degree n.

a) Write a short MATLAB function $(\approx 4\ lines)$ utilizing the Horner's method for evaluating polynomial at a given point x. The first line can be written as follows

%the program starts here

Function y=hornersPoly (p, x)

Where p is the vector of the polynomial coefficient, x is the value where the polynomial is to be evaluated and y is the output value.

(7 Marks)

b) Change your MATLAB function in part (a) allow for vectorised arguments. In other words, suppose that x is now a vector of values where the polynomial is to be evaluated, and y is a vector of outputs. (9 Marks)

c) Use part (a) to find P(3) for the polynomial

$$P(x) = x^5 - 6x^4 + 8x^3 + 4x - 40$$ (4 Marks)

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