# EMBU UNIVERSITY COLLEGE

**(A Constituent College of the University of Nairobi)**

**2015/2016 ACADEMIC YEAR**

**SECOND SEMESTER EXAMINATION**

**FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE**

**SMA 425: PARTIAL DIFFERENTIAL EQUATIONS I**

**DATE: APRIL 11, 2016** **TIME: 11:00-1:00**

**INSTRUCTIONS:**

**Answer Question ONE and ANY Other TWO Questions**

## QUESTION ONE

a) Differentiate between linear and non linear partial differential equations. Give one example for each.

(4 Marks)

b) Eliminate the arbitrary constants from the equation $z = (x^2 + a)(y^2 + b)$ and find the corresponding PDE.

(5 Marks)

c) Show that the direction cosines of the tangent at the point $(x, y, z)$ to the conic $ax^2 + by^2 + cz^2 = 1, x + y + z = 1$ are proportional to $(by - cz, cz - ax, ax - by)$

(4 Marks)

d) Find the general solution of the following partial differential equation

$$px + qy = 3z$$

(4 Marks)

e) Solve the non-linear partial differential equation

$$pq + qx = y$$

(5 Marks)

f) Determine the necessary and sufficient condition of compatibility of two non-linear first order partial differential equations

$$f(x,y,z,p,q)=0$$
$$g(x,y,z,p,q)=0$$

(3 Marks)

g) Find general solution of the partial differential equation

$$(y-z)p+(x-y)q=z-x$$

(5 Marks)

## QUESTION TWO

a) Find the integral curves of the set of equations

$$\frac{dx}{mz-ny}=\frac{dy}{nx-lz}=\frac{dz}{ly-mx}$$

(7 Marks)

b) Find a differential equation arising from

$$\phi(x+y+z,x^2+y^2-z^2)=0$$

(6 Marks)

c) Find the general solution for semi-linear partial differential equation

$$x^2\frac{\partial z}{\partial x}+y^2\frac{\partial z}{\partial y}=(x+y)z$$

(7 Marks)

## QUESTION THREE

a) Find the orthogonal trajectories on the surface $x^2+y^2+2fyz+d=0$ of its curves of intersection with planes parallel to the $xy-plane$.

(10 Marks)

b) Find the partial differential equation arising from $\phi\left(z/x^3, y/x\right)=0$, where $\phi$ is an arbitrary function of the arguments.

(7 Marks)

b) If the rectangular Cartesian coordinates $(x,y,z)$ of a point P in three dimensions are connected by a relation of the form

$$f(x,y,z)=0$$

then point P lies on the surface and the equation is called equation of the surface. Using this equation derive the directional ratios of the normal and tangent to the surface at point P on the surface .

(3 Marks)

**QUESTION FOUR (20 MARKS)-OPTIONAL**

a) Determine the integral surface of the equation

$$z(x+y)p + z(x-y)q = x^2 + y^2$$ (7 Marks)

b) Find the integral curves of the set of equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$ (7 Marks)

c) By eliminating the arbitrary constants $a$ and $b$ , find the partial differential equation arising from the family of spheres of radius 5 with centres on the $x = y$ .

$$(x-a)^2 + (y-a)^2 + (z-b)^2 = 25$$ (6 Marks)

**QUESTION FIVE**

a) Show that the following Pfaffian differential equation is integrable

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

Hence, show that keeping $z$ constant, the integration of the equation yields the following relation.

$$\frac{y(x+z)}{y+z} = F(z)$$ where, $F(z)$ is an arbitrary function of $z$

(10 Marks)

b) Find the general integral surface of the quasi-linear partial differential equation

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$$

(10 Marks)

**--END--**