



EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 409: ORDINARY DIFFERENTIAL EQUATIONS II

DATE: APRIL 7, 2016

TIME: 08:30-10:30AM

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions

QUESTION ONE

a) Determine the general solution of the system of simultaneous differential equations below.

$$\dot{x} - 2x - y = 0$$

$$\dot{y} + 3x - 6y = 0$$

(4 Marks)

b) Solve $2(y + z)dx - (x + z)dy + (2y - x + z)dz = 0$ using the substitution $x = uz$ and $y = vz$.

(5 Marks)

c) Determine whether x^2 , e^x and e^{-x} are linearly independent or linearly dependent.

(5 Marks)

d) Solve the total differential equation $xz^3dx - zdy + 2ydz = 0$.

(4 Marks)

e) Explain what it means to say that a function $f(x)$ is analytic at $x = x_0$.

(2 Marks)

f) Determine the singular points of the differential equation

$$(1 - x^2)y'' - 2xy' + 12y = 0 \text{ and classify them as singular or irregular.}$$

(5 Marks)

g) Express the differential equation $\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 9x = t$ into the matrix equation of the form

$$\vec{x} = A(t)\vec{x}(t) + \vec{f}(t) \text{ and solve the homogeneous part.} \quad (5 \text{ Marks})$$

QUESTION TWO

a) Use the method of Frobenius to find the solution of the differential equation

$$2x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} + (x - 5)y = 0 \quad (13 \text{ Marks})$$

b) Solve the differential equation $yy'' = (y')^2$ (7 Marks)

QUESTION THREE

a) Reduce the differential equation $y''' + 2y'' - y' - 2y = 0$ to a system of first order equations and solve it. (5 Marks)

b) Find $f(y)$ such that $f(y)dx - zxdy - xy \ln y dz = 0$. (5 Marks)

c) Use the integrating factor $\frac{1}{x^2}$ to solve $(2x^3y + 1)dx + x^4dy + x^2 \tan z dz = 0$. (4 Marks)

d) Use the matrix method so solve the system

$$\dot{x} = 3x + 2y + 2z$$

$$\dot{y} = x + 4y + z$$

$$\dot{z} = -2x - 4y - z \quad (6 \text{ Marks})$$

QUESTION FOUR

a) Prove that $dx + dy + ydz = 0$ is not integrable and solve it simultaneously with

$$x - y + z = m. \quad (6 \text{ Marks})$$

b) Solve the non-homogeneous differential equation

$$y'' - 4xy' - 4y = e^x \quad (14 \text{ Marks})$$

QUESTION FIVE

- a) Define the Bessel equation of order two. (1 Mark)
- b) Express $J_3(x)$ in terms of the Bessel function of the lowest possible order. (4 Marks)
- c) Determine the solution to the Legendre differential equation
 $(1 - x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 12y = 0.$ (4 Marks)
- d) i) Define the wronskian of three functions f_1, f_2 and f_3 . (2 Marks)
- ii) Find the wronskian of the functions $f_1 = \sin x$ and $f_2 = \cos x$. (2 Marks)
- e) Solve $y'' \sin y' = \sin x$ which satisfies $y(1) = 2$ and $y'(1) = 1$. (7 Marks)

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