EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

FOURTH YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 401: TOPOLOGY I

DATE: NOVEMBER 30, 2015 **TIME: 8:30- 10:30**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

(a) Let X be a non-empty set. Describe what is meant by the following terms:

i) A metric on X (2 marks)

ii) A topology on X (2 marks)

iii) A finer topology (1 marks)

(b) Consider a function $d: \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ defined by

$$d(x, y) = |x - y|, \quad \forall x, y \in \mathbb{R}$$

Prove that d is a metric on $\mathbb{R}$. (4 marks)

(c) List all topologies on the set $X = \{a, b\}$ (4 marks)

(d) Let X, Y, and Z be topological spaces. Further, let $f: X \to Y$ and $g: Y \to Z$ be functions between the topological spaces.

(i) Describe what it means for the function $f: X \to Y$ to be continuous. (2 marks)

(ii) Prove that if $f: X \to Y$ and $g: Y \to Z$ are continuous functions, then the composite function $(g \circ f): X \to Z$ is continuous. (4 marks)

(e) Consider a topology $\tau = \{\emptyset, X, \{1\}, \{1,2,5\}, \{1,2,3,4\}, \{1,3,4\}\}$ on the set $X = \{1,2,3,4,5\}$ and a subset $A = \{3,4,5\}$ of X. Determine

(i) the derived set of A (3 marks)

(ii) the closure of A (2 marks)

(iii)the interior of A (2 marks)

(iv)the relative topology on A (2 marks)

(f) Describe what is meant by a T_1 topological space (2 marks)

QUESTION TWO

a) Let (X, τ) be a topological space and A a subset of X. Describe what is meant by

(i) the boundary of A (2 marks)

(ii) a relative topology for A (2 marks)

b) Prove that the intersection of any two topologies, τ_1 and τ_2, on a non-empty set X is a topology on X (6 marks)

c) Prove that a relative topology is a topology (10 marks)

QUESTION THREE

a) Describe what it means for a topological space (X, τ) to be

i) connected (2 marks)

ii) compact (3 marks)

b) Let (X, τ) be a topological space, where $X = \{a, b\}$ and $\tau = \{X, \emptyset, \{a\}\}$. Determine whether (X, τ) is connected. (3 marks)

c) Let $f: X \to Y$ be a continuous surjection from topological space X to topological space Y.

i) Prove that if X is connected, then Y is connected (6 marks)

ii) Prove that if X is compact, then Y is compact (6 marks)

QUESTION FOUR

(a) Let $f: (X, \tau_X) \to (Y, \tau_Y)$ be a function from topological space (X, τ_X) to topological space (Y, τ_Y). Describe what it means for f to be

(i) continuous at a point $p \in X$ (2 marks)

(ii) a homeomorphism (2 marks)

(b) Prove that the identity function from a topological space to itself is continuous (3 marks)

(c) Prove that homeomorphism is an equivalence relation over topological spaces (13 marks)

QUESTION FIVE

(a) Let (X, τ) be a topological space. Describe what it means to say that

(i) $\mathfrak{B}$ is a basis for the topology on X (3 marks)

(ii) M is an open neighborhood of $x \in X$ (2 marks)

(iii) X is a Hausdorff space (or X is a T_2- space) (3 marks)

(b) Show that every subspace of a T_2-space is a T_2-space (3 marks)

(c)

(i) Describe what it means for a sequence $\{x_n\}$ in a topological space (X, τ) to converge to a point in X (3 marks)

(ii) Let X be a Hausdorff space. Prove that if $\{x_n\}$ is a sequence in X with $x_n \to x$ and $x_n \to y$, then $x = y$. In other words, prove that limits of sequences in Hausdorff spaces are unique, where they exist. (6 marks)

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