**EMBU UNIVERSITY COLLEGE**

**(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)**

**FIRST SEMESTER EXAMINATIONS 2014/2015**

**THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE**

**SMA 341: PROBABILITY AND STATISTICS II**

**DATE: DECEMBER 11, 2014** **TIME: 08:00 – 10:00AM**

**INSTRUCTIONS:**

**Answer Question ONE and ANY Other TWO Questions.**

## QUESTION ONE

a) Prove that if $A$ is non – countable and $A \subset B$ , then $B$ is also non – countable. (3 marks)

b) i) Define the probability space. (5 marks)

ii) The measure $P$ is $\sigma$ - additive. For events $A_n$ , $n \geq 1$, show that

$$P(\bigcup_{n=1}^{\infty} A_n) \leq \sum_{n=1}^{\infty} P(A_n)$$ (4 marks)

c) Let $X$ be a random variable with probability generating function $G(S)$. Find the generating function of $Y = X + 1$ (4 marks)

d) Let $X_1, X_2, X_3$ and $X_4$ be a random sample of size 4 from the distribution having probability distribution function

$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & e/w \end{cases}$$

If $Y_1 < Y_2 < Y_3 < Y_4$ are the order statistics corresponding to this sample,

i) Find the probability distribution function of $Y_3$ and then (4 marks)

ii) Compute $\Pr(\frac{1}{2} < Y_3)$. (5 marks)

e) Let $\underset{\sim}{x} \sim N(\mu, \Sigma)$, $\underset{\sim}{\mu} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\Sigma = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $\underset{\sim}{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. Let $y_1 = x_1 + x_2$, $y_2 = x_2 + x_3$.

Find the joint distribution of $y_1, y_2$. (5 marks)

## QUESTION TWO

a) Compute an approximate probability that the mean of a random sample of size 20 from a distribution having p.d.f.

$$f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & e/w \end{cases}$$

is between 0.75 and 0.90. (6 marks)

b) Show that $\frac{S(1+S)}{(1-S)^3}$ is the generating function of the sequence

$\{0^2, 1^2, 2^2, 3^2 \cdots\}$ (8 marks)

c) Suppose that $X_1, X_2, X_3$ are independent random variables and that each has a standard normal distribution. Find $P(3X_1 + 2X_2 - 6X_3 < -7)$. (6 marks)

## QUESTION THREE

a) One of the numbers 1, 2, 3, 4, 5 and 6 is to be chosen by casting an unbiased die. Let this random experiment be repeated five independent times. Let the random variable $X_1$ be the number of terminations in the set $\{x; x = 1, 2, 3\}$ and let the random variable $X_2$ be the number of terminations in the set $\{x; x = 4, 5\}$. Find $P\{X_1 = 2; X_2 = 1\}$ (10 marks)

b) Let $\underset{\sim}{x} = (x_1, x_2, x_3)$ be jointly normally distributed as

$f(\underset{\sim}{x}) = ke^{-\frac{1}{2}Q}$ where $Q = 2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_2$. Identify the mean vector $\underset{\sim}{\mu}$, the variance covariance matrix $\Sigma$ and the constant $k$. (10 marks)