



# EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 322: NUMERICAL ANALYSIS I

DATE: APRIL 13, 2016

TIME: 02:00-04:00

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## INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

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### QUESTION ONE

- a) Use examples to distinguish between chopping and rounding off methods of determining the floating point form of a positive real number. (4 Marks)
- b) Evaluate the approximate maximum error in  $7.18 \times 96.1$ . (5 Marks)
- c) Using the nodes  $x_0 = 2, x_1 = 2.5$  and  $x_2 = 4$ , find the second interpolating polynomial for  $f(x) = \frac{1}{x}$ . (5 Marks)
- d) i) Find the approximate value of  $I = \int_0^2 \frac{dx}{1+x}$  using Simpson's rule. (4 Marks)
- ii) Obtain a bound for the errors. The exact value of  $I = \ln 2 = 0.693147$  correct to six decimal places. (2 Marks)
- e) Perform 5 iterations of the secant/chord method to obtain the root of the equation  $\cos x - xe^x = 0$  taking the initial approximations as  $x_0 = 0, x_1 = 1$ . (5 Marks)

f) Calculate  $\int_0^{1/2} \frac{x}{\sin x} dx$  using the trapezoidal rule with  $h = \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$  and Romberg

integration. Assume  $f(0)$  is taken as the limiting value. (5 Marks)

### QUESTION TWO

a) Find the quadrature formula  $\int_0^1 f(x) \frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$  which is exact for polynomials of highest possible degree. (15 Marks)

b) Use the formula in (a) above to evaluate  $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ . (5 Marks)

### QUESTION THREE

a) i) Given  $X$  and  $Y$  are real numbers whose approximations are  $X^x$  and  $Y^x$  respectively, derive an expression for the maximum possible relative error in  $\frac{X^x}{Y^x}$ . (7 Marks)

ii) Hence determine the maximum relative error in  $\frac{16.72}{11.9}$ . (3 Marks)

b) Given that  $f(0) = 1, f(1) = 3, f(3) = 55$ , find :

i) The unique polynomial of degree two or less, which fits the given data. (8 Marks)

ii) The bound on the error. (2 Marks)

### QUESTION FOUR

a) Given that  $f(x) = xe^x$ , calculate the approximate value of  $f'(2.0)$  using:

i) 3 point formula with  $h = 0.1$ . (3 Marks)

ii) 5 point formula with  $h = 0.1$ . (3 Marks)

b) What is the sum of the binary numbers 1101 and 111 in decimal system? (2 Marks)

c) Use Romberg Integration to compute  $R_{61}$  for the function  $\int_0^\pi \sin x dx$  with  $n = 6$ . Compare your answer with the exact value. (8 Marks)

d) For the following data, calculate the differences and obtain the forward and backward difference polynomials. (4 Marks)

$x$	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.40	1.56	1.76	2.00	2.28

**QUESTION FIVE**

a) Use the Newton- Raphson method to approximate the solution of the non-linear system (6 Marks)

$$x^2 + y^2 = 4$$

$$x - y = 0$$

in the 1<sup>st</sup> quadrant  $(x, y)$  given that  $(x_0, y_0) = (1, 1)$  with  $n = 5$

b) The following values of the function  $f(x) = \cos x + \sin x$  are given

$x$	$10^\circ$	$20^\circ$	$30^\circ$
$f(x)$	1.1585	1.2817	1.3660

i) Construct the quadratic interpolating polynomial that fits the data. (8 Marks)

ii) Hence find  $f\left(\frac{\pi}{12}\right)$ . Compare with the exact value. (3 Marks)

c) Given that  $f(x) = x^2 e^x$ , approximate  $f''(3)$  using  $h = 0.1$ . (3 Marks)

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