EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

THIRD YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 302: REAL ANALYSIS II

DATE: APRIL 12, 2016 **TIME: 02:00-04:00**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) i) Let **K** denote a space of real or complex numbers. Distinguish between pointwise convergence and uniform convergence of a sequence of functions $\{f_n\}$ on **K.**

(2 Marks)

ii) Prove that if $f_n(x)$, $g_n(x)$ are sequences of convergent functions of real numbers with limits $f(x)$, $g(x)$ respectively and $f_n(x) \leq g_n(x) \; \forall \; n \in \boldsymbol{N}$ then $f(x) \leq g(x)$

(5 Marks)

b) Prove that if $p < 1$, then the series $\sum_{n \in N} \frac{1}{n^P}$ is divergent. (4 Marks)

c) i) Define a monotonic (increasing) function (1 Mark)

ii) Show that all the monotonic functions on bounded interval are of bounded variation

(5 Marks)

d) Define a Dirichlet function. Hence show that a Dirichlet function on the interval $[a, b]$ is not Riemann Integrable. (4 Marks)

e) Show that for positive numbers x and y, $x^{logy}/y^{logx} = 1$. (2 Marks)

f) Define the Lebesgue Integral of the function f with respect to the measure μ over the set E. (3 Marks)

g) Prove that if the function $f(x)$ is odd on the interval $(-\ell, \ell)$, then $\int_{-\ell}^{\ell} f(x)dx = 0$ (3 Marks)

QUESTION TWO

a) Show that in general absolute convergence of series implies convergence in a metric space $(\boldsymbol{K}, d)$ and by using an appropriate counter example illustrate that the converse need not to be true. (6 Marks)

b) State and prove Comparison test (Weiertrass M-Test) for convergence of series of real valued functions (7 Marks)

c) State the D'Alembert's Ratio Test for convergence of an infinite series. Hence prove that if $l < 1$ in the statement of the test, then the series is convergent. (7 Marks)

QUESTION THREE

a) Discuss the following concepts as used in real analysis

i) A partition of a closed interval $[a, b]$ (1 Mark)

ii) The Riemann's' upper sum and lower sum of the function f (4 Marks)

iii) The Riemann Stieltjes Integrable function on $[a, b]$ (3 Marks)

b) Show that the function $f(x) = x$ is Riemann Integrable in $[0,1]$ and that $\int_0^1 f(x) = \frac{1}{2}$. (8 Marks)

c) Let $f(x) = x$ for $a \leq x \leq b$ and define α on $[a, b]$ by $\alpha x = 0$ for $a \leq x < b$ with $\alpha(b) = c$. If (P, t) is a tagged partition of $[a, b]$ with the partition P, define the Riemann Stieltjes sum by $t_n c$. Show that $\int_a^b x d\alpha(x) = bc$.

(4 Marks)

QUESTION FOUR

a) i) Define a bounded variation function f on a closed real valued interval $[a, b]$.

(2 Marks)

ii) Hence prove that if f is a bounded variation function on $[a, b]$, then f is bounded.

(6 Marks)

b) Sketch graphs of an exponential function $f(x) = a^x$ and a logarithmic function $f(x) = log_a x$ on the same axes ($a > 0, taking\ a = 2$). What relationship exists between the two graphs of functions? Hence state any three differences in these graphs.

(8 Marks)

c) Prove that for $a, b > 0$ $(a, b \neq 1)$, $log_a(x) = \frac{log_b(x)}{log_b(a)}$ (4 Marks)

QUESTION FIVE

a) Define the Fourier series of the function$f(x)$ on the interval $-l\ to\ l$. (5 Marks)

b) Find the Fourier series of the function defined by (10 Marks)

$$f(x) = 0, \quad for - \pi < x < 0, \quad and \qquad f(x) = x \quad for\ 0 < x < \pi.$$

c) List any five evident properties of a Lebesque integrable function (5 Marks)

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