EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE & BACHELOR OF EDUCATION (SCIENCE/ARTS)

SMA 240: PROBABILITY AND STATISTICS I

DATE: APRIL 8, 2016 **TIME: 08:30-10:30AM**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 Marks)

a) Differentiate between

i) k^{th} central moment about the origin and k^{th} central moment about its mean (2 Marks)

ii) Skewness and Kurtosis (3 Marks)

b) The joint p.d.f of X and Y is given by

$$f(x,y)=\begin{cases}2, 0<x<y<1\\0, otherwise\end{cases}$$

Find

i) $f_2(y)$. (2 Marks)

ii) $f(x/y)$ (2 Marks)

iii) $E(X/Y)$. (3 Marks)

iv) $Var(X/Y)$ (4 Marks)

c) Let $X_1, X_2, \cdots, X_{10}$ be a random sample from a distribution that is normally distributed as $n(5,10)$.

Find $P(159.9 < \sum_{i=1}^{10}(x_i - 5)^2 < 251.9)$. (4 Marks)

d) Let X and Y have a bivariate normal distribution with parameters $\mu_1 = 3$, $\mu_2 = 1$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$ and $\rho = 3/5$. Determine

(i) $P(3 < Y < 8 / x = 7)$ (4 Marks)

e) If X_1 and X_2 are random variables such that $\text{var}(X_1) < \infty$ and $\text{var}(X_2) < \infty$, show that $\text{var}(X_1 + X_2) = \text{var}(X_1) + \text{var}(X_2) + 2\text{cov}(X_1 X_2)$ (4 Marks)

f) Let the joint p.d.f of X and Y be

$$f(x,y) = \begin{cases} \dfrac{x+y}{32}, x = 1,2; y = 1,2,3,4 \\ 0, elsewhere \end{cases}$$

Show that $f(x,y)$ is a joint probability function of X and Y (2 Marks)

QUESTION TWO

a) Let $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, elsewhere \end{cases}$ be the p.d.f of X. Find the distribution function and p.d.f of $Y = \sqrt{X}$ (5 Marks)

b) Suppose that X_1, X_2 and X_3 are independent random variables and that each has a standard normal distribution, find $P(3X_1 + 2X_2 - 6X_3 < -7)$ (5 Marks)

c) Let X and Y be two random variables with joint density function given by

$$f(x,y) = \begin{cases} x^2 + \dfrac{xy}{3}, 0 \le x \le 1; 0 \le y \le 2 \\ 0, elsewhere \end{cases}$$

Find the conditional expectation, $E(X \mid Y = 1/2)$ (10 Marks)

QUESTION THREE

a) The joint probability density of the random variables X and Y is given by

$$f(x,y)=\begin{cases}\frac{1}{64}e^{-y/8}, 0\le x\le y<\infty\\ 0, otherwise\end{cases}$$

Find the covariance of X and Y. (8 Marks)

b) Let X be a random variable with p.d.f given by

$$f(x)=\begin{cases}\frac{1}{\beta}e^{-x/\beta}, x>0\\ 0, elsewhere\end{cases}$$

Find the moment generating function, m.g.f, of X (6 Marks)

c) Let X be a random variable and $g(x)$ be a non – negative function with the domain the real line. Show that

$P(g(x)\ge k)\le \frac{E[g(x)]}{k}$ for every $k>0$. (8 Marks)

QUESTION FOUR

a) If X is a random variable such that $E(X)=3$ and $E(X^2)=13$, using Chebyshev's inequality, find an upper bound for $P(|X-3|\ge 5)$. (5 Marks)

b) Suppose that X_1 and X_2 are independent random variables and that the p.d.f of each of these variables is as follows:

$$f(x)=\begin{cases}e^{-x}, x\ge 0\\ 0, elsewhere\end{cases}.$$

i) Find the joint p.d.f of $Y_1=X_1+X_2$ and $Y_2=X_1$ (7 Marks)

ii) Calculate the marginal distributions of both Y_1 and Y_2 (6 Marks)

iii) Are Y_1 and Y_2 independent? (2 Marks)

QUESTION FIVE

a) Given the simple linear regression equation to be

$Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, $i = 1,2,\cdots,n$, show that

i) $\hat{\beta}_0 = \overline{Y} - \hat{\beta}_1 \overline{X}$ and (3 Marks)

ii) $\hat{\beta}_1 = \dfrac{\sum X_i Y_i - n\overline{X}\overline{Y}}{\sum X^2{}_i - n\overline{X}^2}$, by the method of least squares estimates, through the application of matrix algebra. (5 Marks)

b) If X_1, X_2, $\cdots$, X_n are independent random variables each with a Poisson distribution with parameter λ, show that $Y = X_1 + X_2 + \cdots + X_n$ has a Poisson distribution with parameter $n\lambda$. (4 Marks)

c) Find the p.d.f of $Y = X^2$ given the p.d.f of X to be:

$$f(x) = \begin{cases} \frac{1}{6}, x = -2,-1,0,1,2,3 \\ 0, elsewhere \end{cases}$$ (4 Marks)

d) The probability distribution function of X is given by

$$f(x) = \begin{cases} \frac{1}{6}, x = 1,2,3,4,5,6 \\ 0, elsewhere \end{cases}$$

Find

μ_1' and μ_2 (4 Marks)

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