

EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE & BACHELOR OF EDUCATION (SCIENCE/ARTS)

SMA 240: PROBABILITY AND STATISTICS I

DATE: APRIL 8, 2016

TIME: 08:30-10:30AM

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE (30 Marks)

a) Differentiate between

i) k^{th} central moment about the origin and k^{th} central moment about its mean

(2 Marks)

ii) Skewness and Kurtosis

(3 Marks)

b) The joint p.d.f of X and Y is given by

$$f(x,y) = \begin{cases} 2.0 < x < y < 1 \\ 0, otherwise \end{cases}$$

Find

i) $f_2(y)$.

(2 Marks)

ii) f(x/y)

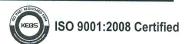
(2 Marks)

iii) E(X/Y).

(3 Marks)

iv) Var(X/Y)

(4 Marks)



c) Let X_1 , X_2 , ..., X_{10} be a random sample from a distribution that is normally distributed as n(5,10).

Find
$$P(159.9 < \sum_{i=1}^{10} (x_i - 5)^2 < 251.9)$$
. (4 Marks)

- d) Let X and Y have a bivariate normal distribution with parameters $\mu_1 = 3$, $\mu_2 = 1$, $\sigma_1^2 = 16$, $\sigma_2^2 = 25$ and $\rho = \frac{3}{5}$. Determine
 - (i) P(3 < Y < 8/x = 7) (4 Marks)
- e) If X_1 and X_2 are random variables such that $var(X_1) < \infty$ and $var(X_2) < \infty$, show that $var(X_1 + X_2) = var(X_1) + var(X_2) + 2 cov(X_1 X_2)$ (4 Marks)
- f) Let the joint p.d.f of X and Y be

$$f(x,y) = \begin{cases} \frac{x+y}{32}, & x = 1,2; y = 1,2,3,4\\ 0, & elsewhere \end{cases}$$

Show that f(x, y) is a joint probability function of X and Y (2 Marks)

QUESTION TWO

- a) Let $f(x) = \begin{cases} 1, 0 < x < 1 \\ 0, elsewhere \end{cases}$ be the p.d.f of X. Find the distribution function and p.d.f of $Y = \sqrt{X}$ (5 Marks)
- b) Suppose that X_1 , X_2 and X_3 are independent random variables and that each has a standard normal distribution, find $P(3X_1 + 2X_2 6X_3 < -7)$ (5 Marks)
- c) Let X and Y be two random variables with joint density function given by

$$f(x,y) = \begin{cases} x^2 + \frac{xy}{3}, 0 \le x \le 1; 0 \le y \le 2\\ 0, elsewhere \end{cases}$$

Find the conditional expectation, $E(X | Y = \frac{1}{2})$ (10 Marks)

QUESTION THREE

a) The joint probability density of the random variables X and Y is given by

$$f(x,y) = \begin{cases} \frac{1}{64} e^{-\frac{y}{8}}, 0 \le x \le y < \infty \\ 0, otherwise \end{cases}$$

Find the covariance of X and Y.

(8 Marks)

b) Let X be a random variable with p.d.f given by

$$f(x) = \begin{cases} \frac{1}{\beta} e^{-x/\beta}, & x > 0\\ 0, & elsewhere \end{cases}$$

Find the moment generating function, m.g.f, of X

(6 Marks)

c) Let X be a random variable and g(x) be a non – negative function with the domain the real line. Show that

$$P(g(x) \ge k) \le \frac{E[g(x)]}{k}$$
 for every $k > 0$. (8 Marks)

QUESTION FOUR

- a) If X is a random variable such that E(X) = 3 and $E(X^2) = 13$, using Chebyshev's inequality, find an upper bound for $P(|X-3| \ge 5)$. (5 Marks)
- b) Suppose that X_1 and X_2 are independent random variables and that the p.d.f of each of these variables is as follows:

$$f(x) = \begin{cases} e^{-x}, x \ge 0\\ 0, elsewhere \end{cases}.$$

- i) Find the joint p.d.f of $Y_1 = X_1 + X_2$ and $Y_2 = X_1$ (7 Marks)
- ii) Calculate the marginal distributions of both Y_1 and Y_2 (6 Marks)
- iii) Are Y_1 and Y_2 independent? (2 Marks)

QUESTION FIVE

a) Given the simple linear regression equation to be

 $Y_i = \beta_0 + \beta_1 X_i + \varepsilon_i$, $i = 1, 2, \dots, n$, show that

- i) $\hat{\beta}_0 = \overline{Y} \hat{\beta}_1 \overline{X}$ and (3 Marks)
- ii) $\hat{\beta}_1 = \frac{\sum X_i Y_i n \overline{XY}}{\sum X_i^2 n \overline{X}^2}, \text{ by the method of least squares estimates, through the}$

application of matrix algebra.

(5 Marks)

- b) If X_1 , X_2 , ..., X_n are independent random variables each with a Poisson distribution with parameter λ , show that $Y = X_1 + X_2 + \dots + X_n$ has a Poisson distribution with parameter $n\lambda$. (4 Marks)
- c) Find the p.d.f of $Y = X^2$ given the p.d.f of X to be:

$$f(x) = \begin{cases} \frac{1}{6}, x = -2, -1, 0, 1, 2, 3\\ 0, elsewhere \end{cases}$$
 (4 Marks)

d) The probability distribution function of X is given by

$$f(x) = \begin{cases} \frac{1}{6}, x = 1,2,3,4,5,6\\ 0, elsewhere \end{cases}$$

Find

 μ_1' and μ_2

(4 Marks)

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