



EMBU UNIVERSITY COLLEGE
(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE,
BACHELOR OF EDUCATION SCIENCE AND BACHELOR OF EDUCATION ARTS

SMA 208: ORDINARY DIFFERENTIAL EQUATIONS 1

DATE: DECEMBER 9, 2015

TIME: 14:00-16:00

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

- a.) Define the following terms
- i). Linearity, degree and order of a differential equation (3 marks)
 - ii). Ordinary differential equation and partial differential equation (2 marks)
- b.) Obtain a differential equation for which $y = A \cos \alpha x + B \sin \alpha x$ is a solution, where A and B are arbitrary constant and α a fixed constant. (4 marks)
- c.) Solve the differential equation (4marks)

$$\frac{dy}{dx} - \frac{y}{x-2} = 2(x-2)^2$$

- d.) Solve the variable separable differential equation (4 marks)

$$\frac{3y^2x}{(1-2y^3)} \frac{dy}{dx} - 1 = 0$$

e.) Show that the differential equation $(3x^2y^2 - y^5)dx + (x^3y^2 - 3xy^4)dy = 0$ is not exact and show how to get integrating factor to make it exact. Hence, or otherwise make it an exact differential equation. (3 marks)

f.) In a certain culture of bacteria the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, how many may be expected at the end of 12 hours? (4 marks)

g.) If you are using method of undetermined coefficients to solve this equation, what is the best form of the trial particular solution?

$$y'' + 9y = 6 \sin 3x \quad \dots\dots\dots (3 \text{ marks})$$

h.) Find the general solution of the given differential equation.

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 15y = 0 \quad \dots\dots\dots (3 \text{ marks})$$

QUESTION TWO

a.) Derive the transformation of the Bernoulli's equation into first order first degree

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

and hence solve the equation

$$\frac{dy}{dx} + 2xy + xy^4 = 0 \quad \dots\dots\dots (7 \text{ marks})$$

b.) Find the solution to the initial value problem

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} - 10\frac{dy}{dx} = 0,$$

$$y(0) = 7, y'(0) = 0, y''(0) = 70$$

(8 marks)

c.) Find the general solution to the linear differential equation given below.

$$\frac{1}{x^2} \frac{dy}{dx} + \frac{3}{x^3} y = \frac{2}{x^5} \quad \dots\dots\dots (5 \text{ marks})$$

QUESTION THREE

- a) Solve the differential equation

$$\frac{dy}{dx} = \frac{4x - 2y + 4}{2x + y - 2} \quad (8 \text{ marks})$$

- b) Given the differential equation, test for exactness and hence solve it.

$$\frac{dy}{dx} = \frac{x + \sin y}{-x \cos y + 2y} \quad (7 \text{ marks})$$

- c) A cup of coffee at $190^\circ F$ is left in a room temperature of $70^\circ F$.

At time $t = 0$, the coffee is cooling at $15^\circ F$ per minute. Assuming that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings.

Find the function that models the cooling of the coffee.

(5marks)

QUESTION FOUR

- a.) Find the general solution to the differential equation

$$(D^2 - 3D + 2)y = \cos x$$

(7 marks)

- b.) Find the solution to the following differential equation

$$y + px = x^4 p^2$$

Where $p = \frac{dy}{dx}$ (6 marks)

c.) Use the power series method to find the general solution of the differential equation.

$$\frac{d^2 y}{dx^2} - y = 0 \quad (7 \text{ marks})$$

QUESTION FIVE

a.) By the use of the method of variation of parameters, obtain the general solution of the equation

$$\frac{d^2 y}{dx^2} + 4y = \sin 2x \sec^2 2x \quad (10 \text{ marks})$$

b.) Use the method of undetermined coefficients to find the solutions of the differential equation.

$$\frac{d^2 y}{dx^2} - 3 \frac{dy}{dx} + 2y = 3e^{-x} - 10 \cos 3x \quad (10 \text{ marks})$$

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