EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 206: INTRODUCTION TO ANALYSIS

DATE: APRIL 14, 2016 **TIME: 11:00-01:00**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Define the following terms:

  i) Open set (2 Marks)

  ii) Closed set (2 Marks)

b) State the completeness axiom. (3 marks)

c) Define a rational number. Show that if $s=\sqrt{n+1}-\sqrt{n-1}$ for any integer $n \geq 1$, then s is irrational. (5 Marks)

d) Let x and y be positive real numbers. Show that:

  i) $x+y$ is also positive (3 Marks)

  ii) $x<y$ if and only if $x^2<y^2$ (3 Marks)

e) Show that a set A is closed if and only if its equal to its closure. (4 Marks)

f) Show that a sequence x_n of real numbers converges to a point x iff every subsequence of x_n converges to x (4 Marks)

g) Let $f:[a,b] \to \mathbf{R}$ be a bounded function. Define the lower and upper Riemann sums of f. (4 Marks)

QUESTION TWO

a) Show that the function

$$f(x)=\begin{cases} 1, & x \text{ is rational} \\ -1, & x \text{ is irrational} \end{cases}$$

Is not Riemann integrable. (6 Marks)

b) State and prove the intermediate value theorem (6 Marks)

c) Define the concept of uniform continuity (2 Marks)

d) Show that $K \subset \mathbf{R}$ is compact, then it has a maximum and minimum. (6 Marks)

QUESTION THREE

a) Let $f:\mathbf{R} \to \mathbf{R}$ be defined by

$$f(x)=\begin{cases} 3-x, & x>1 \\ 1, & x=1 \\ 2x, & x<1 \end{cases}$$

Sketch the graph of f and find $f(1^+)$ and $f(1^-)$. Is f continuous at $x=1$? (7 Marks)

b) Define the limit of a function and show that this limit is unique.(7 Marks)

c) Show that for a sequence (x_n) of real numbers, $x_n \to x$ iff $\overline{\lim}\, x_n = \underline{\lim}\, x_n = x$ (6 Marks)

QUESTION FOUR

a) Show that every convergent sequence or real numbers is bounded.

(7 Marks)

b) Show that the set of all polynomials with integral coefficients is countable.

(7 Marks)

c) Define $d:\mathbf{R}^2 \times \mathbf{R}^2 \to \mathbf{R}$ by $d(x,y) = |x_1 - y_1| + |x_2 - y_2|$ $x = (x_1, x_2)$, $y = (y_1, y_2)$ show that d is a metric on $\mathbf{R}^2$ (6 Marks)

QUESTION FIVE

a) Show that $\sqrt{p}$ is irrational, where p is a prime integer. (7 Marks)

b) State and prove the criterion for Riemann integrability. (10 Marks)

c) Define a Cauchy sequence. (3 Marks)

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