EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION (SCIENCE)

SMA 205: INTRODUCTION TO ALGEBRA

DATE: APRIL 13, 2016 **TIME: 02:00-04:00**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Define the following terms (4 Marks)

i) Binary operation

ii) Prime integer

b) Write the following pairs of integers in the form $a = qb + r$, $0 \leq b \leq r$ (3 Marks)

i) $a = 24$, $b = 11$

ii) $a = 876$, $b = 25$

iii) $a = 464$, $b = 16$

c) Define a relation on the set of integers as follows: $a \sim b$ if and only if $a - b$ is an even integer. Determine if $\sim$ is an equivalence relation. (4 Marks)

d) Define a semigroup. (4 Marks)

e) Let G be a group such that $a^2 = e$. Show that G is abelian (4 Marks)

f) Let G and H be groups and let e' be the identity element of H. Show that the mapping given by $\phi(x) = e'$ is a homomorphism. (3 Marks)

g) Define the center of a group G and show that it is a subgroup. (6 Marks)

h) State the Lagrange theorem. (2 Marks)

QUESTION TWO

a) Prove that every subgroup of a cyclic group is cyclic. (7 Marks)

b) State and prove the division algorithm (10 Marks)

c) Define the greatest common divisor of two integers. (3 Marks)

QUESTION THREE

a) Prove that the following matrices (10 Marks)

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \text{ and } \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Form a multiplicative group.

b) Define an isomorphism between groups and show that if G is a group of positive real numbers under multiplication and H be the additive group of real numbers, the mapping given by $\theta(x) = \log x(base10)$ is an isomorphism. (10 Marks)

QUESTION FOUR

a) Define an equivalence relation and show that the relation $\subseteq$ is not an equivalent relation. (6 Marks)

b) Let * be defined on Q by $a * b = a + b - ab$

   a. Find $7 * \frac{1}{2}$ (2 Marks)

   b. Is $(Q, *)$ a semigroup? (5 Marks)

   c. Is * commutative? (4 Marks)

   d. Find the identity element for * (3 Marks)

QUESTION FIVE

a) Define a field and hence show that the set of real numbers of the form $a + b\sqrt{3}$ where a and b are rational numbers is a field. (9 Marks)

b) Define the following (4 Marks)

i) Integral domain

ii) Division ring

c) Show that if F is a field then its characteristic is either zero or a prime number.

(7 Marks)

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