EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF SCIENCE, BACHELORS OF EDUCATION SCIENCE/ARTS

SMA 203: LINEAR ALGEBRA I

DATE: DECEMBER 8, 2015 **TIME: 14:00-16:00**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Distinguish the following terms as used in linear algebra.
   - i.) A degenerate and a homogenous linear equation (2 marks)
   - ii.) Transpose matrix and symmetric matrix (2 marks)
   - iii.) Linearly dependence and linearly independence vectors (2 marks)
   - iv.) A Subspace and a basis (2 marks)

b) (i) Determine the rank of the following matrix by reducing the matrix to echelon form

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 3 & -1 & 1 \end{pmatrix}$$ (3 marks)

(ii) Illustrate using appropriate examples of matrices A and B, that if $AB = 0$, then it doesn't imply that $A = 0 \; or \; B = 0$ (3 marks)

c) Determine if $S = \{2+x+x^2, x-2x^2, 2+3x-x^2\}$ is linearly independent in P^2 (4 marks)

d) Find the inverse of the following matrix by means of row reduction approach

$$\begin{pmatrix} cos\theta & sin\theta & 0 \\ -sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ (3marks)

e) Evaluate the WROSKIAN, $W(e^x, e^{2x}, e^{5x},\ 0)$ (4 marks)

f) Prove that for any vector $u, v \in \mathbb{R}^n$ and any scalar $a, b, \in \mathbb{R}$.

$$a(u + v) = au + av$$ (2 marks)

g) Prove that for any two bases of a vector space, they have the same number of vectors. (3 marks)

QUESTION TWO

a) Consider the system in unknowns x *and* y

$$\begin{aligned} x - 2y \quad &= 1 \\ x - y + az &= 2 \\ ay + 9z &= b \end{aligned}$$

Find which values of a does the system have a unique solution, and for which pairs of values (a, b) does the system have more than one solution. (7 marks)

b) Define an elementary matrix. (1 mark)

If $A_{(m \times n)}$ is a matrix, prove that the following statements are equivalent

i.) A is invertible

ii.) $A\underline{x} = \underline{b}$ has a unique for any b

iii.) $A\underline{x} = 0$, has a trivial solution

iv.) A is row equivalent to I_n. (12 marks)

QUESTION THREE

a) Solve the following system of equations of the planes by use of Gauss elimination method

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 1 \\ -x_1 + 2x_2 - 3x_3 &= 3 \\ x_1 + 3x_2 - 2x_3 &= 4 \end{aligned}$$

Hence give the geometrical interpretation of the solution of the planes (7 marks)

b) Define the following terms as used in linear transformation

i.) Kernel of T

ii.) Range of T

Hence prove that if $T: U \rightarrow V$ is a linear transformation, then the kernel of T is a subspace of U. (6 marks)

c) (i) Let $V = P_4$ and W be the set of all polynomials of degree 4 or less but with a constant zero term. Determine if W is a subspace of P_4. (3 marks)

(ii) Determine if $T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$ defined as $T(x_1, x_2) = (2x_1,\ x_1 - x_2,\ x_2 + 2x_1)$ is a linear transformation. (4 marks)

QUESTION FOUR:

(a) Determine if $S = \{(1,2,1), (2,9,0), (3,3,4)\}$ is a basis for $\mathbb{R}^3$ (7 marks)

(b) Find the basis and dimension of the solution space for the equations

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$

$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$

$$x_3 + x_4 + x_5 = 0$$

(7 marks)

(c) Let $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be defined as $(x) = \begin{pmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

Find the Basis for $R(T)$ and the Nullity (T). (6 marks)

QUESTION FIVE

a) Find the inverse of the following matrix by first getting the adjoint

$$\begin{pmatrix} 2 & 1 & -2 \\ 3 & 2 & 2 \\ 5 & 4 & 3 \end{pmatrix}$$

Hence or otherwise, solve the following system of linear equations

$$x_1 + x_2 - 2x_3 = 10$$
$$3x_1 + 2x_2 + 2x_3 = 1$$
$$5x_1 + 4x_2 + 3x_3 = 4$$

(8 marks)

b) Find the determinant of matrix A by appropriately first partitioning the matrix

$$A = \begin{pmatrix} 3 & 6 & 9 & 3 \\ -1 & 0 & 1 & 0 \\ 1 & 3 & 2 & -1 \\ -1 & 2 & -1 & 1 \end{pmatrix}$$

(5 marks)

c) Let $V = R^4$ and $S = \{(1,-2,0,3),(2,3,0,-1),(2,-1,2,1)\}$. Determine if $(3,9,-4,-2) \in L(S)$, where $L(S)$ is a subset of V. (7 marks)

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