EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF ECONOMICS

SMA 203: LINEAR ALGEBRA

DATE: APRIL 13, 2016 **TIME: 08:30-10:30**

INSTRUCTIONS: Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Let V be a vector space over $\mathbb{R}$. Describe what is meant by a subspace of V (3 marks)

b) Let S be a subset of $\mathbb{R}^3$ consisting of all vectors of the form $(t, 0, -2t)$, $t \in \mathbb{R}$, together with standard vector addition and scalar multiplication. Determine whether S is a subspace of $\mathbb{R}^3$. (Justify your answer) (4 marks)

c) Express the vector $\boldsymbol{w} = (2,5)$ as a linear combination of $\boldsymbol{u} = (1,2)$ and $\boldsymbol{v} = (3,5)$ (5 marks)

d) Find the determinant of the matrix $M = \begin{bmatrix} 1 & -1 & -2 \\ 2 & -3 & -5 \\ -1 & 3 & 5 \end{bmatrix}$ (5 marks)

e) Determine the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 1 & 1 \\ -2 & 4 \end{bmatrix}$ (6 marks)

f) Using the inverse matrix method, solve the following system of linear equations

$$2x - 3y = 7$$
$$3x + 7y = -1$$
(5 marks)

g) Let V be a vector space over $\mathbb{R}$. Describe what is meant by a bilinear form on V (2 marks)

h)

QUESTION TWO

a) Define the terms linear dependence and linear independence as they apply to the vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_k$ in a vector space V. (4 marks)

b) Determine if the vectors $\boldsymbol{v}_1 = (1,3,-1)$, $\boldsymbol{v}_2 = (2,1,0)$, and $\boldsymbol{v}_3 = (4,2,1)$ are linearly dependent or linearly independent. (7 marks)

c) Show that the vectors $\boldsymbol{v}_1 = (1,1,1)$, $\boldsymbol{v}_2 = (1,1,0)$, and $\boldsymbol{v}_3 = (1,0,1)$ form a basis for $\mathbb{R}^3$. (4 marks)

d) Find a basis for the space spanned by the vectors $\boldsymbol{v}_1 = (1,0,-2,3)$, $\boldsymbol{v}_2 = (0,1,2,3)$, $\boldsymbol{v}_3 = (2,-2,-8,0)$, $\boldsymbol{v}_4 = (2,-1,10,3)$ and $\boldsymbol{v}_5 = (3,-1,-6,9)$ (5 marks)

QUESTION THREE

(a) Define the terms eigenvalue and eigenvector for a matrix A (5 marks)

(b) Compute all the eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} 2 & -2 & 2 \\ 0 & 1 & 1 \\ -4 & 8 & 3 \end{bmatrix}$ (15 marks)

QUESTION FOUR

(a) Define the term linear transformation (3 marks)

(b) Determine whether the function $T: \mathbb{R}^3 \to \mathbb{R}^3$, defined by $T(x, y, z) = (z, x + y, y)$ is a linear transformation. (5 marks)

(c) Let $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 3 \\ 3 & 2 & -2 \end{bmatrix}$. Evaluate:

(i) determinant of A (2 marks)

(ii) adjoint of A (4 marks)

(iii) inverse of A (2 marks)

(d) Use the result in Question 4 (c) to solve the following system of linear equations using the inverse matrix method:

$$x + y - z = 2$$
$$2x - y + 3z = 5$$
$$3x + 2y - 2z = 5$$

(4 marks)

QUESTION FIVE

a) Let $M_{2\times2}$ denote the set of all 2×2 matrices. Further, let a subset S of $M_{2\times2}$ be given by $S = \left\{ \begin{bmatrix} 2x & y \\ 3x + y & 3y \end{bmatrix} : \ x, y \in \mathbb{R} \right\}$. Is S a subspace of $M_{2\times2}$? Explain. (4 marks)

b) Define the following terms:

(i) Symmetric bilinear form on a vector space V (2 marks)

(ii) Quadratic form on $\mathbb{R}$ (3 marks)

c) Determine the symmetric matrix A corresponding to the quadratic form

$$Q(\boldsymbol{x}) = Q(x_1, x_2, x_3) = 3x_1^2 + 4x_1x_2 - x_2^2 + 8x_1x_3 - 6x_2x_3 + x_3^2$$

Verify your result; in other words, show that $Q(\boldsymbol{x}) = \boldsymbol{x}^{\mathrm{T}}\mathrm{A}\boldsymbol{x}$ (6 marks)

d) Consider a function $B: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$, defined as follows:

$$B(\boldsymbol{x}, \boldsymbol{y}) = 2x_1y_2 - 3x_2y_1, \text{for all } \boldsymbol{x} = (x_1, x_2), \boldsymbol{y} = (y_1, y_2) \in \mathbb{R}^2$$

Determine whether B is a bilinear form on $\mathbb{R}^2$. Justify your answer (5 marks)

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