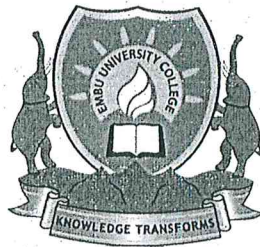


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EMBU UNIVERSITY COLLEGE
(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

SECOND SEMESTER EXAMINATIONS 2013/2014

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF
SCIENCE

SMA 201: ADVANCED CALCULUS

DATE: APRIL 4, 2014

TIME: 8.30 – 10.30AM

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

- a) State the mean value theorem of integral calculus. (2 marks)
- b) A system of particles consists of three masses $m_1=3$, $m_2=4$ and $m_3=2$ at the points with position vectors $r_1=(2,-1,3)$, $r_2=(5,2,4)$ and $r_3=(-2,0,1)$. Find its centre of mass. (4 marks)
- c) Evaluate $\int_0^1 \int_0^1 \frac{dx dy}{\sqrt{(1-x^2)(1-y^2)}}$ (5 marks)

- d) Show that $f(x, y) = 3x - y$ is continuous at $(2, 1)$. (2 marks)
- e) Explain the meaning of the term ‘‘improper integrals’’ and hence prove whether the improper integral $\int_1^{\infty} \frac{dx}{x}$ is convergent or divergent. (5 Marks)
- f) Examine the function $f(x, y) = x^3 + y^3 - 3axy$ for maxima and minima values. (6 Marks)
- g) Use Lagrange multipliers to investigate the extrema of the function $f(x, y) = x^2y$ subject to the constraint $2x^2 + y^2 = 3$. (6 marks)

QUESTION TWO (20 MARKS)

- a) Let $f(x, y) = 24xy - 6x^2y$. Find f_x and f_y and evaluate f_x and f_y at $(1, 2)$. (6 marks)
- b) Given that $f(x, y) = \tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$, determine $\frac{\partial f}{\partial x}$. (4 marks)
- c) Find the moments of inertia about the x- and y- axes of a plate of density $\rho(x, y) = y$, shaped like the region R bounded by the positive coordinates axes and the parabola $y^2 = 1 - x$. (4 marks)
- d) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (\sqrt{a^2 - x^2 - y^2}) dx dy$. (6 marks)

QUESTION THREE (20 MARKS)

- a) i.) Define the Taylor’s series generated by the function f. (2 marks)
- ii.) Find the Taylor series generated by $f(x) = \frac{1}{x}$ at $a=2$. (4 marks)
- (iii) Show that the series is geometric and converges to $\frac{1}{x}$. (5 marks)
- b) Given $f(x, y) = \cos^{-1}\frac{y}{x}$, find f_x+f_y (9 marks)

QUESTION FOUR (20 MARKS)

a) Show that if u is a homogeneous function of degree n in x and y , then

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = n(n-1)u. \quad (12 \text{ marks})$$

b) Find the mean value of the function $f(x) = 1 - x^3$ on the interval $[0,4]$ and show that f takes this value on the interval $[1,3]$. (5 marks)

d) Show that $f(x, y) = \cos x + y \sin y$ is continuous at $(5, 2\pi)$ (3 marks)

QUESTION FIVE (20 MARKS)

a) Examine the maxima and minima values of the function

$$f(x, y) = \sin x + \sin y + \sin(x + y)$$

(10 marks)

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$ (8 marks)

c) State two advantages of the method of Lagrange Multipliers over other methods used in optimization. (2 marks)

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