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EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

SECOND SEMESTER EXAMINATIONS 2013/2014

SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 201: ADVANCED CALCULUS

DATE: APRIL 4, 2014 **TIME: 8.30 – 10.30AM**

INSTRUCTIONS:

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS.

QUESTION ONE (30 MARKS)

a) State the mean value theorem of integral calculus. (2 marks)

b) A system of particles consists of three masses $m_1 = 3$, $m_2 = 4$ and $m_3 = 2$ at the points with position vectors $r_1 = (2,-1,3)$, $r_2 = (5,2,4)$ and $r_3 = (-2,0,1)$. Find its centre of mass. (4 marks)

c) Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{(1-x^2)(1-y^2)}}$ (5 marks)

d) Show that $f(x,y) = 3x - y$ is continuous at (2, 1). (2 marks)

e) Explain the meaning of the term ''improper integrals''and hence prove whether the improper integral $\int_1^{\infty} \frac{dx}{x}$ is convergent or divergent. (5 Marks)

f) Examine the function $f(x,y) = x^3 + y^3 - 3axy$ for maxima and minima values. (6 Marks)

g) Use Lagrange multipliers to investigate the extrema of the function $f(x,y) = x^2y$ subject to the constraint$2x^2 + y^2 = 3$. (6 marks)

QUESTION TWO (20 MARKS)

a) Let$f(x,y) = 24xy - 6x^2y$. Find f_x and f_y and evaluate f_x and f_y at(1,2). (6 marks)

b) Given that $f(x,y) = \tan^{-1}(\frac{x^2+y^2}{x+y})$, determine $\frac{\partial f}{\partial x}$. (4 marks)

c) Find the moments of inertia about the x- and y- axes of a plate of density$\rho(x,y) = y$, shaped like the region R bounded by the positive coordinates axes and the parabola $y^2 = 1 - x$.

(4 marks)

d) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (\sqrt{a^2 - x^2 - y^2})dxdy$. (6 marks)

QUESTION THREE (20 MARKS)

a) i.) Define the Taylor's series generated by the function f. (2 marks)

ii.) Find the Taylor series generated by $f(x) = \frac{1}{x}$ at a=2. (4 marks)

(iii) Show that the series is geometric and converges to $\frac{1}{x}$. (5 marks)

b) Given $f(x,y) = \cos^{-1}\frac{y}{x}$, find f_x+f_y (9 marks)

QUESTION FOUR (20 MARKS)

a) Show that if u is a homogeneous function of degree n in x and y, then

$x^2 \frac{\partial 2_u}{\partial x^2} + 2xy \frac{\partial 2_u}{\partial x \partial y} + y^2 \frac{\partial 2_u}{\partial y^2} = n\,(n-1)\,u.$ (12 marks)

b) Find the mean value of the function $f(x) = 1 - x^3$ on the interval [0,4] and show that f takes this value on the interval[1,3]. (5 marks)

d) Show that $f(x, y) = \cos x + y \sin y$ is continuous at $(5, 2\pi)$ (3 marks)

QUESTION FIVE (20 MARKS)

a) Examine the maxima and minima values of the function

$f(x, y) = \sin x + \sin y + \sin(x + y)$

(10 marks)

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ (8 marks)

c) State two advantages of the method of Lagrange Multipliers over other methods used in optimization. (2 marks)

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