EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

FIRST SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 201: ADVANCED CALCULUS

DATE: DECEMBER 10, 2015 **TIME: 8:30-10:30**

INSTRUCTIONS:
Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Classify according to the type of the improper fraction (2 marks)

i.) $\int_0^{\pi} \frac{1-\cos x dx}{x^2}$

ii.) $\int_0^{\infty} \frac{e^{-x} dx}{\sqrt{x}}$

b) Evaluate

$$\frac{d}{dx}\int_x^{x^2} \frac{\sin t}{t} dt$$ (3 marks)

c) Investigate the convergence/divergence of the following integral (5 marks)

$$\int_{-1}^{7} \frac{dx}{\sqrt[3]{x+1}}$$

d) Show that the function

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2} & (x,y)\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

is continuous function at every point except at the origin. (5 marks)

e) If $U = z\sin\frac{y}{x}$ where $x = 3r^2 + 2s$, $y = 4r - 2s^2$, and $z = 2r^2 - 3s^2$.

Find $\frac{\partial U}{\partial r}$ (5 marks)

f) Suppose R is a triangular region between the graphs of $y = 1 - x$, $y = 0$, $x = 1$ *and* $x = -2$ and that $f(x, y) = 4 - y$ in R

Evaluate

$$\iint_R f(x,y)dA$$ (5 marks)

g) Evaluate

$$\int_{x=0}^{3}\int_{y=0}^{1}\int_{z=0}^{2}(x+2y-z)dxdydz$$ (5 marks)

QUESTION TWO

a) If $\phi(\alpha)=\int_{\alpha}^{\alpha^2}\frac{\sin\alpha x dx}{x}$

Find $\phi'(\alpha)$ where $\alpha \neq 0$

(Use Leibnitz's rule) (6 marks)

b) Find the minimum value of the function $F(x, y) = x^2 + y^2$ subject to the constraint

$x^2 + 8xy + 7y^2 = 225$

(8 marks)

c) State and prove Rolle's Theorem of the mean. (6 marks)

QUESTION THREE

a) The surfaces $F(x,y,z)=x^2+y^2=0,\ and\ G(x,y,z)=x+z-4=0$ meet in an ellipse E. Find

i.) the parametric equations of the **line tangent** to E at point P(1,1,3). (5 marks)

ii.) the equation of the **plane normal** to E at the point P(1,1,3) (3 marks)

d) Find the relative maxima and minima of the function

$$f(x,y)=x^3+y^3-3x-12y+20$$ (8 marks)

c) Let R be a rectangular region bounded by the lines $x=-1,\quad x=2,\quad y=0,\quad \text{and}\quad y=2$

Find

$$\iint_R x^2 y\,dA$$ (4 marks)

QUESTION FOUR

a) A region R in the xy plane bounded by $x+y=6,\quad x-y=2,\quad y=0$

i) Determine the region R^* in the uv plane into which R is mapped under the transformation $x=u+v,\quad y=u-v$.

ii) Compute the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$

iii) Compare the results of (ii) with the ratio of areas of R and R^*. (8 marks)

b) Determine the directional derivative of

$f(x,y,z)=2xy-z$ at (2, -1, 1) in the direction of (3, 1, -1) (6 marks)

Show that (6 marks)

$$\lim_{x\to 0^+}\frac{In\cos 3x}{In\cos 2x}=\frac{9}{4}$$

QUESTION FIVE

a) A thin plate covers a region bounded by the $x-axis$ and the lines $x=1,$ *and* $y=2x$ in the first quadrant.
The plates density at the point $\rho(x,y)=6x+6y+6$
Find the plates

i). Mass
ii). Moments of inertia about the x-axis.

(10 marks)

b) Verify Green's Theorem in the plane for

$$\oint_C (2xy-x^2)dx+(x+y^2)dy$$

Where C is the closed curve of the region bounded by $y=x^2$ and $y^2=x$, and the two curves intersects at points $(0,0)$ and $(1,1)$.

(10 marks)

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