EMBU UNIVERSITY COLLEGE

(A CONSTITUENT COLLEGE OF THE UNIVERSITY OF NAIROBI)

FIRST SEMESTER EXAMINATIONS 2014/2015

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE

SMA 201: ADVANCED CALCULUS

DATE: DECEMBER 16, 2014 **TIME: 13:30 – 15:30**

INSTRUCTIONS:

Answer Question ONE and ANY Other TWO Questions.

QUESTION ONE

a) Show that $f(x,y) = 3x - 2y$ is continuous at (2, 4). (2 marks)

b) Prove whether the improper integral $\int_1^\infty \frac{dx}{x}$ is convergent or divergent. (3 marks)

c) If $x = r\cos\propto$ and $y = r\sin\propto$, determine $\frac{\partial(x,y)}{\partial(r,\propto)}$. (3 marks)

d) Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3axy$ (5 marks)

e) Use Lagrange multipliers to investigate the extrema of the function $f(x,y) = x^2y$ subject to the constraint $2x^2 + y^2 = 3$. (4 marks)

f) Evaluate $\int_0^1 \int_0^2 (9x^2 + 3y^2 + 2)dxdy$ (4 marks)

(g) If $f(x,y) = \tan^{-1}\frac{y}{x}$, verify that $f_{xx} + f_{yy} = 0$ (5 marks)

(h) A system of particles consists of three masses $m_1 = 3$, $m_2 = 4$ and $m_3 = 2$ at the points with position vectors $r_1 = (2,-1,3)$, $r_2 = (5,2,4)$ and $r_3 = (-2,0,1)$. Find its centre of mass. (4 marks)

QUESTION TWO

a) Given $x^3z^2 - 5xy^5z = x^2 + y^3$, determine $\frac{\partial z}{\partial y}$. (3 marks)

b) Find the moments of inertia about the x- and y- axes of a plate of density $\rho(x, y) = y$, shaped like the region R bounded by the positive coordinates axes and the parabola $y^2 = 1 - x$.

(5 marks)

c) i) Distinguish between Taylor's series and Maclaurin's series. (2 marks)

ii) Find the Taylor series generated by $f(x) = \frac{1}{x}$ at a = 2. (5 marks)

iii) Show that the series is geometric and converges to $\frac{1}{x}$. (5 marks)

QUESTION THREE

a) Given $u = \sin^{-1}(x - y), x = 3t, y = 4t^3$, show that $\frac{du}{dt} = \frac{3}{\sqrt{1-t^2}}$. (7 marks)

b) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dzdydx}{\sqrt{1-x^2-y^2-z^2}}$ (8 marks)

c) Given $u = x + y^2$, $v = y + z^2$ and $w = z + x^2$, prove that $\frac{\partial x}{\partial u} = \frac{1}{1+8xyz}$.

(5 marks)

QUESTION FOUR

a) Given that $z = tan^{-1}(\frac{x^2+y^2}{x+y})$, determine $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}$. (7 marks)

b) If u is a homogeneous function of degree n in x and y, show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x \partial y}+y^2\frac{\partial^2 u}{\partial y^2}=n(n-1)u$$ (9 marks)

(c) Find the mean value of the function $f(x) = 1 - x^3$ on the interval $[0,4]$ and show that f takes this value on the interval$[0,4]$. (4 marks)

QUESTION FIVE

a) Examine the maxima and minima values of the function $f(x,x) = sinx + siny + \sin(x + y)$. (6 marks)

b) Use Lagrange multipliers to find the greatest area that can be enclosed by a fence of length $800m$. (4 marks)

c) Use Green's Theorem to evaluate the line integral $\int_c ((2x^2 - y^2)dx + (x^2 + y^2)dy)$ where c is the boundary in the xy plane of the area enclosed by the $x - axis\ and\ the\ semi - circle$

$x^2 + y^2 = 1$ in the upper half of the plane. (10 marks)

--END--