EMBU UNIVERSITY COLLEGE

(A Constituent College of the University of Nairobi)

2015/2016 ACADEMIC YEAR

SECOND SEMESTER EXAMINATION

SECOND YEAR EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE, BACHELOR OF EDUCATION (SCIENCE/ARTS), BACHELOR OF SCIENCE (INDUSTRIAL CHEMISTRY), BACHELOR OF SCIENCE (COMPUTER SCIENCE) AND BACHELOR OF SCIENCE (STATISTICS)

SMA 140/ STA 101/ CSC 124: INTRODUCTION TO PROBABILITY AND STATISTICS

DATE: APRIL6, 2016 **TIME: 08:30-10:30**

INSTRUCTIONS:

Answer Question ONE and ANY other two Questions

QUESTION ONE

a) Outline three main characteristics of the standard deviation. (3 Marks)

b) Explain briefly the term kurtosis. (4 Marks)

c) Show that the following function is a probability density function.

i) $f(x)=\begin{cases} e^{-x}, 0<x<\infty \\ 0, elsewhere \end{cases}$ (2 Marks)

d) Find the constant k so that the following is a p.d.f

$$f(x)=\begin{cases} kx^2(1-x), 0<x<1 \\ 0, elsewhere \end{cases}$$ (2 Marks)

Hence find $P(\frac{1}{2}<x<\frac{2}{3})$. (2 Marks)

e) Differentiate between **expectation** and **moment** of a random variable x. (4 Marks)

f) If X is a random variable, then show that $E[X-E(X)]^2 = E(X^2)-[E(X)]^2$ (3 Marks)

g) The data below are the number of seeds germinated out of five planted seeds in each of the 50 pots.

No. of seeds germinated (X)	1	2	3	4	5
No. of pots (f)	8	16	14	9	3

Find the geometric mean of the number of seeds germinated. (4 Marks)

h) The joint probability distribution function (p.d.f) of X and Y is given by

$$f(x,y) = \begin{cases} 6x^2y, & 0 \le x < 1, 0 < y < 1 \\ 0, & elsewhere \end{cases},$$

i) Find $f_1(x)$ and $f_2(y)$ (3 Marks)

ii) Find μ_x and σ_x^2 (3 Marks)

QUESTION TWO

a) Assume $S = A_1 \cup A_2 \cup \cdots \cup A_n$, where $P(A_i) > 0$, $i = 1,2,3,\cdots,n$ and $A_i \cap A_j = \Phi$ for $i \neq j$. Then for any event B, with $P(B) > 0$, show that:

$$P(A_j \mid B) = \frac{P(A_j)P(B \mid A_j)}{\sum_{i=1}^{n} P(A_i)P(B \mid A_i)}$$ (4 Marks)

b) Find the mean and the variance of the following distributions:

(i) $f(x) = \begin{cases} x/6, x = 1,2,3 \\ 0, e/w \end{cases}$ (4 Marks)

(ii) $f(x) = \begin{cases} 6x(1-x), 0 < x < 1 \\ 0, e/w \end{cases}$ (5 Marks)

c) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{x+y}{21}, x = 1,2,3; y = 1,2 \\ 0, e/w \end{cases}$$

i) Show that $f(x,y)$ is a joint p.d.f of x and y. (3 Marks)

ii) Is X and Y independent? (4 Marks)

QUESTION THREE

a) Suppose a statistics class contains 70% male and 30% female students. It is known that in a test, 5% of males and 10% of females got an "A" grade. If one student from this class is randomly selected and observed to have an "A" grade, what is the probability that this is a male student? (8 Marks)

b) The data below gives the Marks scored by 90 students in the Statistics class in Embu University College.

Marks	10-20	20-30	30-40	40-50	50-60	60-70	70-80	80-90	90-100
No. of Students	4	10	A	B	16	C	7	3	1

Given the lower quartile and the mode to be 35 and 44 respectively,

(i) Find the values of A, B and C (2 Marks)

(ii) Estimate the median and standard deviation (3 Marks)

(iii) Estimate the 3^{rd} decile and the 20^{th} percentile. (3 Marks)

(iv) Calculate Karl Pearson's Coefficient of skewness (3 Marks)

Hence comment on the skewness of the distribution (1 Mark)

QUESTION FOUR

a) Suppose that a fair coin is tossed thrice. Let X be the number of heads.

i) Find the sample space (1Mark)

ii) Find the probability function for x. (2 Marks)

iii) Find the cumulative distribution function of x. (3 Marks)

iv) In different graphs, sketch both the probability function and the cumulative distribution function. (4 Marks)

b) Let X be a Poisson distributed random variable. Show that the

i) $E(X) = \lambda$, (3 Marks)

ii) $Var(X) = \lambda$ and (3 Marks)

iii) $M_x(t) = e^{\lambda(e^t - 1)}$ (4 Marks)

QUESTION FIVE

a) Let X be a uniformly distributed random variable on (a,b). Show that the

i) $E(X)=\frac{a+b}{2}$, (3 Marks)

ii) $Var(X)=\frac{(b-a)^2}{12}$ and (3 Marks)

iii) $M_x(t)=\frac{e^{tb}-e^{ta}}{t(b-a)}$ (3 Marks)

b) Using moments, calculate a measure of relative skewness and a measure of relative kurtosis for the following distribution and comment on the results obtained. (11 Marks)

Daily wages (shs.)	No. of workers
70 – 90	8
90 – 110	11
110 – 130	18
130 – 150	9
150 – 170	4

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